Online Appendix to Valuation Risk Revalued*

Oliver de Groot   Alexander W. Richter   Nathaniel A. Throckmorton

*de Groot, University of Liverpool Management School & CEPR, Chatham Street, Liverpool, L69 7ZH, UK (oliverdegroot@gmail.com); Richter, Research Department, Federal Reserve Bank of Dallas, 2200 N. Pearl Street, Dallas, TX 75201 (alex.richter@dal.frb.org); Throckmorton, Department of Economics, William & Mary, P.O. Box 8795, Williamsburg, VA 23187 (nat@wm.edu). The views in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

1 Indifference Curve Derivation

For compactness, define $\rho = 1 - 1/\psi$ and $\alpha = 1 - \gamma$. Then from (7) and (8) in the main paper

$$\bar{U}_C(a_{t+1}) \equiv g(U^C_{t+1}) = g\left((1-\beta+a_{t+1}\beta\bar{x})^{1/\rho}\right),$$
$$\bar{U}_R(a_{t+1}) \equiv g(U^R_{t+1}) = g\left((1-a_{t+1}\beta+a_{t+1}\beta\bar{x})^{1/\rho}\right),$$

where $g(U_{t+1}) = (E_t[U^\alpha_{t+1}])^{1/\alpha}$. The certainty equivalent is given by

$$\bar{U} = (1-\beta+\beta\bar{x})^{1/\rho}.$$

Suppose there are two possible outcomes for a_{t+1}, denoted a_1 and a_2. Then

$$\bar{U}_C = \left(\frac{(1-\beta+a_1\beta\bar{x})^{\alpha/\rho}+(1-\beta+a_2\beta\bar{x})^{\alpha/\rho}}{2}\right)^{1/\alpha},$$
$$\bar{U}_R = \left(\frac{(1-a_1\beta+a_1\beta\bar{x})^{\alpha/\rho}+(1-a_2\beta+a_2\beta\bar{x})^{\alpha/\rho}}{2}\right)^{1/\alpha}.$$

Set $\bar{U}_C$ and $\bar{U}_R$ equal to the certainty equivalent, fix a_1, and solve for a_2 to obtain:

$$a^C_2 = \frac{\left(2\bar{U}^\alpha - (1-\beta+a_1\beta\bar{x})^{\alpha/\rho}\right)^{\rho/\alpha} - (1-\beta)}{\beta\bar{x}},$$
$$a^R_2 = \frac{\left(2\bar{U}^\alpha - (1-a_1\beta+a_1\beta\bar{x})^{\alpha/\rho}\right)^{\rho/\alpha} - 1}{\beta(\bar{x}-1)}.$$

We plot combinations of (a_1, a_2) under the current and revised preferences.

2 Isomorphic Representations of the Current Specification

In the current literature, the preference shock typically hits current utility. If, for simplicity, we abstract from Epstein-Zin preferences, then the utility function and Euler equation are given by

$$U_t = \alpha_t u(c_t) + \beta E_t[U_{t+1}], \tag{2.1}$$
$$\beta E_t[(\alpha_{t+1}/\alpha_t)u'(c_{t+1})/u'(c_t)r_{y,t+1}] = 1. \tag{2.2}$$

The shock follows $\Delta\hat{\alpha}_{t+1} = \rho\Delta\hat{\alpha}_t + \sigma_\alpha\varepsilon_t$, so the change in α_t is known at time t. Alternatively, if the preference shock hits future consumption, the utility function and Euler equation are given by

$$U_t = u(c_t) + a_t\beta E_t[U_{t+1}], \tag{2.3}$$
$$a_t\beta E_t[u'(c_{t+1})/u'(c_t)r_{y,t+1}] = 1. \tag{2.4}$$

If the shock follows $\hat{a}_t = \rho\hat{a}_{t-1} + \sigma_a\varepsilon_t$, the two specifications are isomorphic because setting $a_t \equiv \alpha_{t+1}/\alpha_t$ in (2.4) yields (2.2). We use the second specification because it is easier to compare the current and revised preferences when the shock always shows up in the Euler equation in levels.

3 RESULT 5 PROOF

The results in this section apply a variant of the following four limits:

1. $\lim_{\epsilon\to 0}\left[\frac{1}{\epsilon^2}\left(\left(E\left[(x\epsilon^2 c+1)^{1/\epsilon}\right]\right)^{\epsilon}-1\right)\right] = E[cx]$
2. $\lim_{\epsilon\to 0}\left[\frac{1}{\epsilon^2}\left(\left(E\left[(x\epsilon^2 c+x)^{1/\epsilon}\right]\right)^{\epsilon}-E[x]\right)\right]$ is undefined
3. $\lim_{\epsilon\to 0}\left[\frac{1}{\epsilon}\left(\left(E\left[(x\epsilon c+1)^{1+\epsilon}\right]\right)^{\frac{1}{1+\epsilon}}-1\right)\right] = E[cx]$
4. $\lim_{\epsilon\to 0}\left[\frac{1}{\epsilon}\left(\left(E\left[(x\epsilon c+x)^{1+\epsilon}\right]\right)^{\frac{1}{1+\epsilon}}-E[x]\right)\right] = E[cx]+\mathcal{O}$

where x is an exogenous stochastic variable, c is a stochastic policy relevant variable, and $\mathcal{O} = E[x\log x] - E[x]\log(E[x])$ is an additive term that is independent of the policy relevant variable.

Case 1 Define $\gamma = 1-\epsilon$ and $1-1/\psi = \epsilon^2$. Then preferences are given by

$$U_t^j = \left(w_{1,t}^j c_t^{\epsilon^2} + w_{2,t}^j\left(E_t\left[(U_{t+1}^j)^{\epsilon}\right]\right)^{\epsilon}\right)^{1/\epsilon^2}.$$

For simplicity, assume $a_{t+j}=1$ and c_{t+j} is nonstochastic for $j\geq 2$. Defining $V_t^j = (U_t^j)^{\epsilon^2}$ implies

$$V_t^j = w_{1,t}^j c_t^{\epsilon^2} + w_{2,t}^j\left(E_t\left[(V_{t+1}^j)^{1/\epsilon}\right]\right)^{\epsilon},$$
$$V_{t+1}^j = \textstyle\sum_{k=1}^{\infty}\left(\prod_{i=1}^{k-1} w_{2,t+i}^j\right) w_{1,t+k}^j c_{t+k}^{\epsilon^2}.$$

Combining these results then implies

$$V_t^j = w_{1,t}^j c_t^{\epsilon^2} + w_{2,t}^j\left(E_t\left[\left(\textstyle\sum_{k=1}^{\infty}\tilde{w}_{2,t+k}^j w_{1,t+k}^j c_{t+k}^{\epsilon^2}\right)^{1/\epsilon}\right]\right)^{\epsilon},$$

where $\tilde{w}_{2,t+k}^j \equiv \prod_{i=1}^{k-1} w_{2,t+i}^j$. Now define $W_t^j = (V_t^j-1)/\epsilon^2$, so the utility function is given by

$$W_t = w_{1,t}^j u_t + w_{2,t}^j\left(E_t\left[\left(\tfrac{1}{\epsilon^2}\textstyle\sum_{k=1}^{\infty}\tilde{w}_{2,t+k}^j w_{1,t+k}^j(\epsilon^2 u_{t+k}+1)\right)^{1/\epsilon}\right]\right)^{\epsilon} + \frac{w_{1,t}^j - 1}{\epsilon^2},$$

where $u_t = (c_t^{\epsilon^2}-1)/\epsilon^2$ is a CRRA utility function that converges to $\log c_t$ as $\epsilon\to 0$.

Under the revised specification, $w_{1,t}^R = 1-a_t\beta$ and $w_{2,t}^R = a_t\beta$. Therefore,

$$W_t = (1-a_t\beta)u_t + \frac{a_t\beta}{\epsilon^2}\left(\left(E_t\left[\left(\epsilon^2\textstyle\sum_{k=1}^{\infty}\tilde{a}_{t+k}(1-a_{t+k}\beta)u_{t+k}+1\right)^{1/\epsilon}\right]\right)^{\epsilon}-1\right), \quad (3.1)$$

where $\tilde{a}_{t+k} \equiv \prod_{i=1}^{k-1} a_{t+i}\beta$. Applying Limit 1, then implies

$$\lim_{\epsilon\to 0} W_t = (1-a_t\beta)\log c_t + a_t\beta E_t[\textstyle\sum_{k=1}^{\infty} \tilde{a}_{t+k}(1-a_{t+k}\beta)\log c_{t+k}].$$

Under the current preferences, $w_{1,t}^C = 1-\beta$ and $w_{2,t}^C = a_t\beta$. Therefore,

$$W_t = (1-\beta)u_t + \tfrac{a_t\beta}{\epsilon^2}\left(\left(E_t\left[\left(\epsilon^2(1-\beta)\textstyle\sum_{k=1}^{\infty}\tilde{a}_{t+k}u_{t+k} + 1-\beta+a_{t+1}\beta\right)^{1/\epsilon}\right]\right)^{\epsilon} - \tfrac{1}{a_t}\right),$$

which does not converge to a log utility function as $\epsilon \to 0$ according to Limit 2.

Case 2 The assumption that $\gamma = 1-\epsilon$ and $1-1/\psi = \epsilon^2$ may appear contrived. What is important is that both γ and ψ tend to 1, but ψ approaches 1 at a faster rate. When they approach 1 at the same rate, then time-separable log utility results regardless of whether the preference specification.

To see this result, suppose $\gamma = 1-\epsilon$ and $\psi = 1+\epsilon$. Then utility is given by

$$U_t^j = \left(w_{1,t}^j c_t^{\frac{\epsilon}{1+\epsilon}} + w_{2,t}^j\left(E_t\left[(U_{t+1}^j)^{\epsilon}\right]\right)^{\frac{1}{1+\epsilon}}\right)^{\frac{1+\epsilon}{\epsilon}}.$$

Once again, assume $a_{t+j} = 1$ and c_{t+j} is nonstochastic for $j \geq 2$. Defining $V_t^j = (U_t^j)^{\frac{\epsilon}{1+\epsilon}}$ implies

$$V_t^j = w_{1,t}^j c_t^{\frac{\epsilon}{1+\epsilon}} + w_{2,t}^j\left(E_t\left[\left(\textstyle\sum_{k=1}^{\infty}\tilde{w}_{2,t+k}^j w_{1,t+k}^j c_{t+k}^{\frac{\epsilon}{1+\epsilon}}\right)^{1+\epsilon}\right]\right)^{\frac{1}{1+\epsilon}},$$

where $\tilde{w}_{2,t+k}$ is the same as Case 1. Define $W_t^j = (1+\epsilon)(V_t^j - 1)/\epsilon$. The utility function is given by

$$W_t = w_{1,t}^j u_t + w_{2,t}^j\left(E_t\left[\left(\tfrac{1+\epsilon}{\epsilon}\textstyle\sum_{k=1}^{\infty}\tilde{w}_{2,t+k}^j w_{1,t+k}^j(\tfrac{\epsilon}{1+\epsilon}u_{t+k}+1)\right)^{1+\epsilon}\right]\right)^{\frac{1}{1+\epsilon}} + \left(\tfrac{1+\epsilon}{\epsilon}\right)(w_{1,t}^j - 1),$$

where $u_t = (c_t^{\epsilon/(1+\epsilon)} - 1)/(\epsilon/(1+\epsilon))$ is a CRRA utility function that converges to $\log c_t$ as $\epsilon \to 0$.

Under the revised specification, $w_{1,t}^R = 1-a_t\beta$ and $w_{2,t}^R = a_t\beta$. Therefore,

$$W_t = (1-a_t\beta)u_t + a_t\beta\left(\tfrac{1+\epsilon}{\epsilon}\right)\left(\left(E_t\left[\left(\tfrac{\epsilon}{1+\epsilon}\textstyle\sum_{k=1}^{\infty}\tilde{a}_{t+k}(1-a_{t+k}\beta)u_{t+k}+1\right)^{1+\epsilon}\right]\right)^{\frac{1}{1+\epsilon}} - 1\right),$$

where $\tilde{a}_{t+k}$ is defined above. Applying Limit 3, then implies (3.1).

Under the current preferences, $w_{1,t}^C = 1-\beta$ and $w_{2,t}^C = a_t\beta$. Therefore,

$$W_t = (1-\beta)u_t + a_t\beta\left(\tfrac{1+\epsilon}{\epsilon}\right)\left(\left(E_t\left[\left(\tfrac{\epsilon}{1+\epsilon}(1-\beta)\textstyle\sum_{k=1}^{\infty}\tilde{a}_{t+k}u_{t+k} + a_{t+1}^s\right)^{1+\epsilon}\right]\right)^{\frac{1}{1+\epsilon}} - E_t[a_{t+1}^s]\right)$$
$$+ a_t\beta\left(\tfrac{1+\epsilon}{\epsilon}\right)(E_t[a_{t+1}^s] - 1/a_t),$$

where $a^s_{t+1} \equiv 1 - \beta + a_{t+1}\beta$. Applying Limit 4, then implies

$$\lim_{\epsilon \to 0} W_t = (1-\beta)\log c_t + a_t\beta(1-\beta)E_t[\textstyle\sum_{k=1}^{\infty} \tilde{a}_{t+k} \log c_{t+k}] + \mathcal{O}_t.$$

where $\mathcal{O}_t = E_t[a^s_{t+1} \log a^s_{t+1}] - E_t[a^s_{t+1}] \log(E_t[a^s_{t+1}]) + a_t\beta(E_t[a^s_{t+1}] - 1/a_t) \lim_{\epsilon \to 0} \left(\frac{1+\epsilon}{\epsilon}\right)$ is an exogenous additive term that does not affect the household's optimality conditions.

4 Nonlinear Model Asymptote

Assuming $\mu_{t+1} \equiv y_{t+1}/y_t = d_{t+1}/d_t$, the (nonlinear) Euler equation is given by

$$z_t = \frac{a_t\beta}{1-\chi^j a_t\beta}\left(E_t\Big[\underbrace{\Big(\big(1-\chi^j a_{t+1}\beta\big)\,\mu_{t+1}^{1-1/\psi}(1+z_{t+1})\Big)^{\theta}}_{x_{t+1}}\Big]\right)^{1/\theta}, \tag{4.1}$$

where $\chi^C = 0$ and $\chi^R = 1$. Notice the asymptote disappears if $\mathrm{SD}(x_{t+1}) \to 0$ as $\psi \to 1$. The paper focuses on results from a Campbell and Shiller (1988) approximation of the model. In this appendix, we demonstrate three noteworthy results using the model's exact, nonlinear, form.

One, consider the case without valuation risk, so $a_t = 1$ for all t. The Euler equation reduces to

$$z_t = \beta(E_t[(\mu_{t+1}^{1-1/\psi}(1+z_{t+1}))^{\theta}])^{1/\theta}. \tag{4.2}$$

When $\psi = 1$, we guess and verify that $z_t = \beta/(1-\beta)$, so the price-dividend ratio is constant. This is the well know result that when the IES is 1, the income and substitution effects of a change in endowment growth offset. Therefore, the price-dividend ratio does not respond to cash flow risk.

Two, consider the case when a_t is stochastic under the revised preferences ($\chi^R = 1$) and either $\psi = 1$ (CRRA preferences) or $\mu_t = 1$ for all t (no cash-flow growth). In both cases, we guess and verify that $z_t = a_t\beta/(1-a_t\beta)$. The price dividend ratio is time-varying but independent of θ, so an asymptote does not affect equilibrium outcomes. Thus, the household is certainty-equivalent.

Three, consider what happens under the current preferences ($\chi^C = 0$), which do not account for the offsetting movements in $1-a_t\beta$. To obtain a closed-form solution for any IES, we assume $\mu_t = \mu$ and the preference shock evolves according to $\log(1 + a_{t+1}\eta) = \sigma\varepsilon_{t+1}$, where ε_{t+1} is standard normal. Under these assumptions, we guess and verify that the price-dividend ratio is given by

$$z_t = a_t\eta = a_t\beta\mu^{1-1/\psi}\exp(\theta\sigma^2/2). \tag{4.3}$$

In this case, θ appears in the price-dividend ratio, so the asymptote affects equilibrium outcomes. These results prove that the asymptote is not due to a Campbell-Shiller approximation of the model.

5 ANALYTICAL DERIVATIONS

Stochastic Discount Factor The Lagrangian for specification $j \in \{C, R\}$ is given by

$$U_t^j = \max \left[w_{1,t}^j c_t^{1-1/\psi} + w_{2,t}^j \left(E_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}$$
$$- \lambda_t (c_t + p_{y,t} s_{1,t} + p_{d,t} s_{2,t} - (p_{y,t} + y_t) s_{1,t-1} - (p_{d,t} + d_t) s_{2,t-1}),$$

where $w_{1,t}^C = 1-\beta$, $w_{1,t}^R = 1 - a_t^R \beta$, $w_{2,t}^C = a_t^C \beta$, and $w_{2,t}^R = a_t^R \beta$. The optimality conditions imply

$$w_{1,t}^j \left(U_t^j \right)^{1/\psi} c_t^{-1/\psi} = \lambda_t, \tag{5.1}$$

$$w_{2,t}^j \left(U_t^j \right)^{1/\psi} \left(E_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right] \right)^{\frac{1/\psi-\gamma}{1-\gamma}} E_t \left[\left(U_{t+1}^j \right)^{-\gamma} \left(\partial U_{t+1}^j / \partial s_{1,t} \right) \right] = \lambda_t p_{y,t}, \tag{5.2}$$

$$w_{2,t}^j \left(U_t^j \right)^{1/\psi} \left(E_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right] \right)^{\frac{1/\psi-\gamma}{1-\gamma}} E_t \left[\left(U_{t+1}^j \right)^{-\gamma} \left(\partial U_{t+1}^j / \partial s_{2,t} \right) \right] = \lambda_t p_{d,t}, \tag{5.3}$$

where $\partial U_t^j / \partial s_{1,t-1} = \lambda_t (p_{y,t} + y_t)$ and $\partial U_t^j / \partial s_{2,t-1} = \lambda_t (p_{d,t} + d_t)$ by the envelope theorem. Updating the envelope conditions and combining (5.1)-(5.3) generates (11) and (12) in the paper.

Following Epstein and Zin (1991), we posit the following minimum state variable solution:

$$U_t^j = \xi_{1,t} s_{1,t-1} + \xi_{2,t} s_{2,t-1} \quad \text{and} \quad c_t = \xi_{3,t} s_{1,t-1} + \xi_{4,t} s_{2,t-1}. \tag{5.4}$$

where ξ is a vector of unknown coefficients. The envelope conditions combined with (5.1) imply

$$\xi_{1,t} = w_{1,t}^j (U_t^j)^{1/\psi} c_t^{-1/\psi} (p_{y,t} + y_t), \tag{5.5}$$

$$\xi_{2,t} = w_{1,t}^j (U_t^j)^{1/\psi} c_t^{-1/\psi} (p_{d,t} + d_t). \tag{5.6}$$

Multiplying (5.5) by $s_{1,t-1}$ and (5.6) by $s_{2,t-1}$ and then adding yields

$$U_t^j = w_{1,t}^j (U_t^j)^{1/\psi} c_t^{-1/\psi} ((p_{y,t} + y_t) s_{1,t-1} + (p_{d,t} + d_t) s_{2,t-1}), \tag{5.7}$$

which, after plugging in the budget constraint and imposing equilibrium, can be written as

$$(U_t^j)^{1-1/\psi} = w_{1,t}^j c_t^{-1/\psi} (c_t + p_{y,t} s_{1,t} + p_{d,t} s_{2,t}) = w_{1,t}^j c_t^{-1/\psi} (c_t + p_{y,t}). \tag{5.8}$$

Imposing (5.8) on the utility function implies

$$w_{1,t}^j c_t^{-1/\psi} p_{y,t} = w_{2,t}^j \left(E_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}}. \tag{5.9}$$

Solving (5.8) for U_t^j and (5.9) for $E_t[(U_{t+1}^j)^{1-\gamma}]$ and then plugging into (13) and (14) in the paper implies

$$m_{t+1}^j = (x_t^j)^\theta (c_{t+1}/c_t)^{-\theta/\psi} r_{y,t+1}^{\theta-1}, \tag{5.10}$$

where $x_t^j \equiv w_{2t}^j w_{1t+1}^j / w_{1t}^j$. Taking logs of (5.10) yields (17) in the paper, where

$$\hat{x}_t^C = \hat{\beta} + \hat{a}_t^C,$$
$$\hat{x}_t^R = \hat{\beta} + \hat{a}_t^R + \log(1 - \beta \exp(\hat{a}_{t+1}^R)) - \log(1 - \beta \exp(\hat{a}_t^R)) \approx \hat{\beta} + (\hat{a}_t^R - \beta \hat{a}_{t+1}^R)/(1-\beta),$$

and $\hat{a}_t \equiv \hat{a}_t^C = \hat{a}_t^R/(1-\beta)$ so the preference shocks with the current and revised specifications are directly comparable. It follows that $\hat{x}_t^j = \hat{\beta} + \hat{a}_t - \omega^j \hat{a}_{t+1}$, where $\omega^C = 0$ and $\omega^R = \beta$.

Campbell-Shiller Approximation The return on the endowment is approximated by

$$\begin{aligned}
\hat{r}_{y,t+1} &= \log(p_{y,t+1} + y_{t+1}) - \log(p_{y,t}) \\
&= \log(y_{t+1}(p_{y,t+1}/y_{t+1}) + y_{t+1}) - \log(y_t(p_{y,t}/y_t)) \\
&= \log(y_{t+1}(\exp(\hat{z}_{y,t+1}) + 1)) - \hat{z}_{y,t} - \log(y_t) \\
&= \log(\exp(\hat{z}_{y,t+1}) + 1) - \hat{z}_{y,t} + \Delta \hat{y}_{t+1} \\
&\approx \log(\exp(\hat{z}_y) + 1) + \exp(\hat{z}_y)(\hat{z}_{y,t+1} - \hat{z}_y)/(1 + \exp(\hat{z}_y)) - \hat{z}_{y,t} + \Delta \hat{y}_{t+1} \\
&= \kappa_{y0} + \kappa_{y1} \hat{z}_{y,t+1} - \hat{z}_{y,t} + \Delta \hat{y}_{t+1}.
\end{aligned}$$

The derivation for the equity return, $\hat{r}_{d,t+1}$, is analogous to the return on the endowment.

Model Solution We use a guess and verify method. For the endowment claim, we obtain

$$\begin{aligned}
0 &= \log(E_t[\exp(\hat{m}_{t+1} + \hat{r}_{y,t+1})]) \\
&= \log(E_t[\exp(\theta\hat{\beta} + \theta(\hat{a}_t - \omega^j \hat{a}_{t+1}) + \theta(1 - 1/\psi)\Delta \hat{y}_{t+1} + \theta(\kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t}))]) \\
&= \log\left(E_t\left[\exp\left(\begin{array}{c} \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j \hat{a}_{t+1}) + \theta(1 - 1/\psi)(\mu_y + \sigma_y \varepsilon_{y,t+1}) \\ +\theta\kappa_{y0} + \theta\kappa_{y1}(\eta_{y0} + \eta_{y1}\hat{a}_{t+1}) - \theta(\eta_{y0} + \eta_{y1}\hat{a}_t) \end{array}\right)\right]\right) \\
&= \log\left(E_t\left[\exp\left(\begin{array}{c} \theta\hat{\beta} + \theta(1 - 1/\psi)\mu_y + \theta(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) \\ +\theta(1 - \omega^j \rho_a + \eta_{y1}(\kappa_{y1}\rho_a - 1))\hat{a}_t \\ +\theta(1 - 1/\psi)\sigma_y \varepsilon_{y,t+1} + \theta(\kappa_{y1}\eta_{y1} - \omega^j)\sigma_a \varepsilon_{a,t+1} \end{array}\right)\right]\right).
\end{aligned}$$

This simplifies to

$$\begin{aligned}
0 = \theta\hat{\beta} + \theta(1 - 1/\psi)\mu_y + \theta(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + \tfrac{\theta^2}{2}(1 - 1/\psi)^2 \sigma_y^2 \\
+ \tfrac{\theta^2}{2}(\kappa_{y1}\eta_{y1} - \omega^j)^2 \sigma_a^2 + \theta(1 - \omega^j \rho_a + \eta_{y1}(\kappa_{y1}\rho_a - 1))\hat{a}_t,
\end{aligned}$$

given the log-normality of $\exp(\varepsilon_{y,t+1})$ and $\exp(\varepsilon_{a,t+1})$.

After equating coefficients, we obtain the following exclusion restrictions:

$$\hat{\beta} + (1 - 1/\psi)\mu_y + (\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + \tfrac{\theta}{2}((1 - 1/\psi)^2\sigma_y^2 + (\kappa_{y1}\eta_{y1} - \omega^j)^2\sigma_a^2) = 0, \quad (5.11)$$
$$1 - \omega^j\rho_a + \eta_{y1}(\kappa_{y1}\rho_a - 1) = 0. \quad (5.12)$$

For the dividend claim, we obtain

$$\begin{aligned}
0 &= \log(E_t[\exp(\hat{m}_{t+1} + \hat{r}_{d,t+1})]) \\
&= \log\left(E_t\left[\exp\left(\begin{array}{c} \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j\hat{a}_{t+1}) + (\theta(1-1/\psi) - 1)\Delta\hat{y}_{t+1} + \Delta\hat{d}_{t+1} \\ +(\theta-1)(\kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t}) + (\kappa_{d0} + \kappa_{d1}\hat{z}_{d,t+1} - \hat{z}_{d,t}) \end{array}\right)\right]\right) \\
&= \log\left(E_t\left[\exp\left(\begin{array}{c} \theta\hat{\beta} + (\theta(1-1/\psi) - 1)\mu_y + \mu_d \\ +(\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + (\kappa_{d0} + \eta_{d0}(\kappa_{d1} - 1)) \\ +(\theta(1 - \omega^j\rho_a) + (\theta-1)\eta_{y1}(\kappa_{y1}\rho_a - 1) + \eta_{d1}(\kappa_{d1}\rho_a - 1))\hat{a}_t \\ (\pi_{dy} - \gamma)\sigma_y\varepsilon_{y,t+1} + ((\theta-1)\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1} - \theta\omega^j)\sigma_a\varepsilon_{a,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1} \end{array}\right)\right]\right) \\
&= \theta\hat{\beta} + (\theta(1-1/\psi) - 1)\mu_y + \mu_d + (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + (\kappa_{d0} + \eta_{d0}(\kappa_{d1} - 1)) \\
&\quad + (\theta(1 - \omega^j\rho_a) + (\theta-1)\eta_{y1}(\kappa_{y1}\rho_a - 1) + \eta_{d1}(\kappa_{d1}\rho_a - 1))\hat{a}_t \\
&\quad + \tfrac{1}{2}((\pi_{dy} - \gamma)^2\sigma_y^2 + ((\theta-1)\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1} - \theta\omega^j)^2\sigma_a^2 + \psi_d^2\sigma_y^2).
\end{aligned}$$

Once again, equating coefficients implies the following exclusion restrictions:

$$\begin{aligned}
\theta\hat{\beta} + (\theta(1-1/\psi) - 1)\mu_y + \mu_d + (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + (\kappa_{d0} + \eta_{d0}(\kappa_{d1} - 1)) \\
+\tfrac{1}{2}((\pi_{dy} - \gamma)^2\sigma_y^2 + ((\theta-1)\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1} - \theta\omega^j)^2\sigma_a^2 + \psi_d^2\sigma_y^2) = 0, \quad (5.13)
\end{aligned}$$
$$\theta(1 - \omega^j\rho_a) + (\theta-1)\eta_{y1}(\kappa_{y1}\rho_a - 1) + \eta_{d1}(\kappa_{d1}\rho_a - 1) = 0. \quad (5.14)$$

Equations (5.11)-(5.14) and (21)-(22) in the paper form a system of 8 equations and 8 unknowns.

Asset Prices Given the coefficients, we can solve for the risk free rate. The Euler equation implies

$$\hat{r}_{f,t} = -\log(E_t[\exp(\hat{m}_{t+1})]) = -E_t[\hat{m}_{t+1}] - \tfrac{1}{2}\,\mathrm{Var}_t[\hat{m}_{t+1}],$$

since the risk-free rate is known at time-t. The pricing kernel is given by

$$\begin{aligned}
\hat{m}_{t+1} &= \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j\hat{a}_{t+1}) - (\theta/\psi)\Delta\hat{y}_{t+1} + (\theta-1)\hat{r}_{y,t+1} \\
&= \theta\hat{\beta} + \theta(\hat{a}_t - \omega^j\hat{a}_{t+1}) - \gamma\Delta\hat{y}_{t+1} + (\theta-1)(\kappa_{y0} + \kappa_{y1}\hat{z}_{y,t+1} - \hat{z}_{y,t}) \\
&= \theta\hat{\beta} - \gamma\mu_y + (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + (\theta(1 - \omega^j\rho_a) + (\theta-1)\eta_{y1}(\kappa_{y1}\rho_a - 1))\hat{a}_t \\
&\qquad + ((\theta-1)\kappa_{y1}\eta_{y1} - \theta\omega^j)\sigma_a\varepsilon_{a,t+1} - \gamma\sigma_y\varepsilon_{y,t+1} \\
&= \theta\hat{\beta} - \gamma\mu_y + (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + (1 - \omega^j\rho_a)\hat{a}_t \\
&\qquad + ((\theta-1)\kappa_{y1}\eta_{y1} - \theta\omega^j)\sigma_a\varepsilon_{a,t+1} - \gamma\sigma_y\varepsilon_{y,t+1},
\end{aligned}$$

where the last line follows from imposing (5.12).

Therefore, the risk-free rate is given by

$$\begin{aligned}\hat{r}_{f,t} &= \gamma\mu_y - \theta\hat{\beta} - (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1}-1)) - (1-\omega^j\rho_a)\hat{a}_t \\ &\quad - \tfrac{1}{2}\gamma^2\sigma_y^2 - \tfrac{1}{2}((\theta-1)\kappa_{y1}\eta_{y1} - \theta\omega^j)^2\sigma_a^2.\end{aligned}$$

After plugging in (5.11), we obtain

$$\hat{r}_{f,t} = \mu_y/\psi - \hat{\beta} - (1-\omega^j\rho_a)\hat{a}_t + \tfrac{1}{2}((\theta-1)\kappa_{y1}^2\eta_{y1}^2 - \theta(\omega^j)^2)\sigma_a^2 + \tfrac{1}{2}((1/\psi-\gamma)(1-\gamma)-\gamma)^2\sigma_y^2.$$

Therefore, the unconditional expected risk-free rate is given by

$$E[\hat{r}_f] = -\hat{\beta} + \mu_y/\psi + \tfrac{1}{2}((\theta-1)\kappa_{y1}^2\eta_{y1}^2 - \theta(\omega^j)^2)\sigma_a^2 + \tfrac{1}{2}((1/\psi-\gamma)(1-\gamma)-\gamma)^2\sigma_y^2. \quad (5.15)$$

We can also derive an expression for the equity premium, $E_t[ep_{t+1}]$, which given by

$$\log(E_t[\exp(\hat{r}_{d,t+1} - \hat{r}_{f,t})]) = E_t[\hat{r}_{d,t+1}] - \hat{r}_{f,t} + \tfrac{1}{2}\operatorname{Var}_t[\hat{r}_{d,t+1}] = -\operatorname{Cov}_t[\hat{m}_{t+1}, \hat{r}_{d,t+1}],$$

where the last equality stems from the Euler equation, $E_t[\hat{m}_{t+1}+\hat{r}_{d,t+1}]+\frac{1}{2}\operatorname{Var}_t[\hat{m}_{t+1}+\hat{r}_{d,t+1}] = 0$. We already solved for the SDF, so the last step is to solve for the equity return, which given by

$$\begin{aligned}\hat{r}_{d,t+1} &= \kappa_{d0} + \kappa_{d1}\hat{z}_{d,t+1} - \hat{z}_{d,t} + \Delta\hat{d}_{t+1} \\ &= \kappa_{d0} + \kappa_{d1}(\eta_{d0} + \eta_{d1}\hat{a}_{t+1}) - (\eta_{d0} + \eta_{d1}\hat{a}_t) + \Delta\hat{d}_{t+1} \\ &= \mu_d + \kappa_{d0} + \eta_{d0}(\kappa_{d1}-1) + \eta_{d1}(\kappa_{d1}\rho_a - 1)\hat{a}_t + \kappa_{d1}\eta_{d1}\sigma_a\varepsilon_{a,t+1} + \pi_{dy}\sigma_y\varepsilon_{y,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1}.\end{aligned}$$

Therefore, the unconditional equity premium can be written as

$$E[ep] = \gamma\pi_{dy}\sigma_y^2 + (\theta\omega^j + (1-\theta)\kappa_{y1}\eta_{y1})\kappa_{d1}\eta_{d1}\sigma_a^2. \quad (5.16)$$

To better understand these results, we also note that

$$\begin{aligned}\hat{r}_{d,t+1} - E_t\hat{r}_{d,t+1} &= \kappa_{d1}\eta_{d1}\sigma_a\varepsilon_{a,t+1} + \pi_{dy}\sigma_y\varepsilon_{y,t+1} + \psi_d\sigma_y\varepsilon_{d,t+1}, \\ m_{t+1} - E_t m_{t+1} &= \lambda_a\sigma_a\varepsilon_{a,t+1} + \lambda_y\sigma_y\varepsilon_{y,t+1},\end{aligned}$$

where $\lambda_a \equiv (\theta-1)\kappa_{y1}\eta_{y1} - \theta\omega^j$ and $\lambda_y \equiv -\gamma$ are the market prices for valuation and cash-flow risk.

Long-term Bond Prices The pricing kernel can be written as

$$\hat{m}_{t+1} = m_0 + m_1\hat{a}_t + m_2\sigma_a\varepsilon_{a,t+1} + m_3\sigma_y\varepsilon_{y,t+1},$$

where

$$m_0 \equiv \theta\hat{\beta} - \gamma\mu_y + (\theta-1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1}-1)), \qquad m_2 \equiv (\theta-1)\kappa_{y1}\eta_{y1} - \theta\omega^j,$$
$$m_1 \equiv 1 - \omega^j\rho_a, \qquad m_3 \equiv -\gamma.$$

The 1-period bond price is given by

$$\hat{p}_t^{(1)} = -\hat{r}_{f,t} = \log(E_t[\exp(\hat{m}_{t+1})]) = m_0 + m_1\hat{a}_t + m_2^2\sigma_a^2/2 + m_3^2\sigma_y^2/2.$$

The 2-period bond price is given by

$$\begin{aligned}\hat{p}_t^{(2)} &= \log E_t[\exp(\hat{m}_{t+1} + \hat{p}_{t+1}^{(1)})] \\ &= \log E_t[\exp(m_0 + m_1\hat{a}_t + m_2\sigma_a\varepsilon_{a,t+1} + m_3\sigma_y\varepsilon_{y,t+1} + \\ &\qquad m_0 + m_1(\rho_a\hat{a}_t + \sigma_a\varepsilon_{a,t+1}) + m_2^2\sigma_a^2/2 + m_3^2\sigma_y^2/2)] \\ &= 2m_0 + m_1(1+\rho_a)\hat{a}_t + (m_2+m_1)^2\sigma_a^2/2 + m_2^2\sigma_a^2/2 + m_3^2\sigma_y^2.\end{aligned}$$

More generally, the price of any n-period bond for $n > 1$ is given by

$$\hat{p}_t^{(n)} = nm_0 + m_1\textstyle\sum_{j=0}^{n-1}\rho_a^j\hat{a}_t + \frac{1}{2}\sum_{k=2}^{n}(m_2 + m_1\sum_{j=0}^{n-k}\rho_a^j)^2\sigma_a^2 + \frac{1}{2}m_2^2\sigma_a^2 + \frac{n}{2}m_3^2\sigma_y^2$$

and the risk-free return is given by $r_{f,t}^{(n)} = -\hat{p}_t^{(n)}/n$.

5.1 SPECIAL CASE 1 ($\sigma_a = \psi_d = 0$ & $\pi_{dy} = 1$) In this case, there is no valuation risk ($\hat{a}_t = 0$) and cash flow risk is perfectly correlated ($\Delta\hat{y}_{t+1} = \mu_y + \sigma_y\varepsilon_{y,t+1}$; $\Delta\hat{d}_{t+1} = \mu_d + \sigma_y\varepsilon_{y,t+1}$). Under these assumptions, it is easy to see that (5.15) and (5.16) reduce to (26) and (27) in the paper.

5.2 SPECIAL CASE 2 ($\sigma_y = 0$, $\rho_a = 0$, & $\mu_y = \mu_d$) In this case, there is no cash flow risk ($\Delta\hat{y}_{t+1} = \Delta\hat{d}_{t+1} = \mu_y$) and preference shocks are *i.i.d.* ($\hat{a}_{t+1} = \sigma_a\varepsilon_{a,t+1}$). Under these two assumptions, the return on the endowment and dividend claims are identical, so $\{\kappa_{y0}, \kappa_{y1}, \eta_{y0}, \eta_{y1}\} = \{\kappa_{d0}, \kappa_{d1}, \eta_{d0}, \eta_{d1}\} \equiv \{\kappa_0, \kappa_1, \eta_0, \eta_1\}$. Therefore, (5.15) and (5.16) reduce to (30) and (31) in the paper for the current specification and (32) and (33) in the paper for the revised specification.

The exclusion restriction, (5.12), implies $\eta_1 = 1$ so (5.11) simplifies to

$$0 = \hat{\beta} + (1 - 1/\psi)\mu_y + \kappa_0 + \eta_0(\kappa_1 - 1) + \tfrac{\theta}{2}(\kappa_1 - \omega^j)^2\sigma_a^2. \tag{5.17}$$

First, recall that $0 < \kappa_1 < 1$. Therefore, the asymptote in θ will permeate the solution with the current preferences ($\omega^C = 0$). However, with the revised preferences ($\omega^R = \beta$), we guess and verify that $\kappa_1 = \beta$ when $\psi = 1$. In this case, (5.17) reduces to $\hat{\beta} + \kappa_0 + \eta_0(\beta - 1) = 0$. Combining with (21) in the paper, this restriction implies that $\eta_0 = \log\beta - \log(1-\beta)$ and $\kappa_0 = -(1-\beta)\log(1-\beta) - \beta\log\beta$. Plugging the expressions for η_0, κ_0, and κ_1 into (5.17) and (21) in the paper verifies our initial guess for κ_1.

5.3 PREFERENCE SHOCK ON CURRENT UTILITY

Suppose preferences are instead given by

$$U_t = \left[a_t(1-\beta)c_t^{1-1/\psi} + \beta \left(E_t\left[(U_{t+1})^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \quad 1 \neq \psi > 0. \tag{5.18}$$

Since $w_{1,t} = a_t(1-\beta)$, $w_{2,t} = \beta$, and $x_t = \beta a_{t+1}/a_t$, the SDF is given by

$$\hat{m}_{t+1} = \theta \log \beta + \theta(\hat{a}_{t+1} - \hat{a}_t) - (\theta/\psi)\Delta\hat{c}_{t+1} + (\theta-1)\hat{r}_{y,t+1}. \tag{5.19}$$

Given this slight modification, the average risk-free rate and average equity premium are given by

$$E[\hat{r}_f] = -\log\beta + \mu_y/\psi + ((\theta-1)\kappa_1^2\eta_1^2 + \theta)\sigma_a^2/2, \tag{5.20}$$

$$E[ep] = ((1-\theta)\kappa_1\eta_1 - \theta)\kappa_1\eta_1\sigma_a^2. \tag{5.21}$$

Since $\eta_1 = -1$, there is once again no endogenous mechanism that prevents the asymptote in θ from influencing asset pricing moments, just like in (30) and (31) under the current specification.

5.4 RESULT 7 PROOF

We guess and verify the solution under the revised preferences with $\psi = 1$ is well defined and no asymptote exists. Combine (5.11), (5.12), and (21) in the paper to obtain

$$\begin{aligned} 0 &= \hat{\beta} + (1-1/\psi)\mu_y + \log(1+\exp(\eta_{y0})) - \eta_{y0} \\ &\quad + (1-\gamma)(1-1/\psi)\frac{\sigma_y^2}{2} + \theta\left(\frac{\exp(\eta_{y0})(1-\beta)-\beta}{\exp(\eta_{y0})(1-\rho_a)+1}\right)\frac{\sigma_a^2}{2}. \end{aligned} \tag{5.22}$$

Guess $\eta_{y0} = \log\beta - \log(1-\beta)$, which ensures the last term in parentheses is zero when $\psi = 1$. If the guess is correct then $\hat{\beta} + \log(1+\exp(\eta_{y0})) - \eta_{y0} = 0$. It is straightforward to verify that rearranging this expression returns the original guess. As a result, for $\psi = 1$, the complete solution is given by $\kappa_{y1} = \beta$, $\eta_{y1} = 1$, and $\kappa_{y0} = -\beta\log\beta - (1-\beta)\log(1-\beta)$. Turning to the dividend claim, (5.13) and (5.14) reduce to $\beta = \kappa_{d1}\eta_{d1}$ and $\eta_{d1} = (1-\beta\rho_a)/(1-\kappa_{d1}\rho_a)$, respectively. As a result, $\kappa_{d0} = \kappa_{y0}$, $\kappa_{d1} = \kappa_{y1}$, $\eta_{d0} = \eta_{y0}$, and $\eta_{d1} = \eta_{y1}$. Note that the solution of the model at $\psi = 1$ and the proposition is independent of the values of γ, μ_y, μ_d, σ_a, σ_y, σ_d, π_{dy}, and ρ_a.

5.5 CORRELATED TASTE AND TIME-PREFERENCE SHOCKS

Following Maurer (2012), we introduce two different valuation risk shocks. In this case, the preference specification is given by

$$U_t^M = \left[(1-a_t^T\beta)c_t^{1-1/\psi} + a_t^{TP}\beta\left(E_t\left[(U_{t+1}^M)^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}},$$

where a_t^T is referred to as a "taste" shock and a_t^{TP} is referred to as a "time-preference" shock. In this case, $\hat{x}_t^M = \hat{\beta} + \hat{a}_t^{TP} - \frac{\beta}{1-\beta}(\hat{a}_{t+1}^T - \hat{a}_t^T)$. For simplicity we use the same parameters as Special Case 2. In addition, $\hat{a}_{t+1}^{TP} = \sigma_{TP}\varepsilon_{TP,t+1}$ and $\hat{a}_{t+1}^T = \sigma_T\varepsilon_{T,t+1} + \rho\sigma_{TP}\varepsilon_{TP,t+1}$. Under the Current, Alternative, and Revised preference specifications, $\{\sigma_{TP}, \sigma_T, \rho\}$ equals $\{\sigma_a, 0, 0\}$, $\{0, \sigma_a, 0\}$, and $\{\sigma_a, 0, 1\}$, respectively. Next, we guess and verify the following solution for the price dividend ratio: $\hat{z}_t = \eta_0 + \eta_{TP}\hat{a}_t^{TP} + \eta_T\hat{a}_t^T$. The analogous exclusion restriction to (5.11) is given by

$$\hat{\beta} + \left(1 - \tfrac{1}{\psi}\right)\mu + (\kappa_0 + \eta_0(\kappa_1 - 1)) + \tfrac{\theta}{2}\left(\left(\kappa_1\left(\tfrac{1-\beta\rho}{1-\beta}\right) - \tfrac{\beta\rho}{1-\beta}\right)^2\sigma_{TP}^2 + (\kappa_1 - 1)^2\left(\tfrac{\beta\sigma_T}{1-\beta}\right)^2\right) = 0,$$

since $\eta_{TP} = 1$ and $\eta_T = \beta/(1-\beta)$. The mean risk-free rate and mean equity premium are given by

$$E[\hat{r}_f] = -\hat{\beta} + \mu/\psi + \tfrac{1}{2}\left((\theta - 1)\kappa_1^2\left(1 + \tfrac{\beta\rho}{1-\beta}\right) - \theta\tfrac{\beta\rho}{1-\beta}\right)^2\sigma_{TP}^2,$$
$$E[ep] = \left(\theta\tfrac{\beta\rho}{1-\beta} + (1-\theta)\kappa_1\left(1 + \tfrac{\beta\rho}{1-\beta}\right)\right)\kappa_1\left(1 + \tfrac{\beta\rho}{1-\beta}\right)\sigma_{TP}^2 + (\theta + (1-\theta)\kappa_1)\kappa_1\left(\tfrac{\beta\sigma_T}{1-\beta}\right)^2.$$

It follows that when $\sigma_T = 0$ and $\rho = 0$, we recover the (asymptote afflicted) asset pricing implications of the Current preferences. However, when $\sigma_{TP} = \sigma_a/(1-\beta)$ and ρ approaches 1, so the shock are perfectly correlated, we recover the asset pricing implications of the Revised preferences.

6 Estimation Method

The estimation procedure has two stages. The first stage estimates moments in the data using a 2-step Generalized Method of Moments (GMM) estimator with a Newey and West (1987) weighting matrix with 10 lags. The second stage is a Simulated Method of Moments (SMM) procedure that searches for a parameter vector that minimizes the distance between the GMM estimates in the data and short-sample predictions of the model, weighted by the diagonal of the GMM estimate of the variance-covariance matrix. The second stage is repeated for many different draws of shocks to obtain standard errors for each parameter. The following steps outline the estimation algorithm:

1. Use GMM to estimate the moments, $\hat{\Psi}_T^D$, and the diagonal of the covariance matrix, $\hat{\Sigma}_T^D$.
2. Use SMM to estimate the structural asset pricing model. Given a random seed, s, draw a T-period sequence of shocks for each shock in the model. Denote the shock matrix $\mathcal{E}_T^s$ (e.g., in the baseline model $\mathcal{E}_T^s = [\varepsilon_{y,t}^s, \varepsilon_{d,t}^s, \varepsilon_{a,t}^s]_{t=1}^T$). For $s \in \{1, \ldots, N_s\}$, run the following steps:
   (a) Evaluate the loss function for $i \in \{1, \ldots, N_m\}$ random draws in the parameter space.
      i. Draw $\hat{\theta}_i$ from a multivariate normal distribution centered at a user-specified mean parameter vector, $\bar{\theta}$, with diagonal covariance matrix, Σ_0.
      ii. Solve the structural asset pricing model given $\hat{\theta}_i$. Return to step i if the nonlinear solver (`csolve`) fails to find the unknown coefficients (i.e., solve the model).

iii. Given $\mathcal{E}^s(r)$, simulate the model R times for T periods. For each repetition r, calculate the moments $\Psi_T^M(\hat{\theta}_i, \mathcal{E}^s(r))$.

iv. Calculate the mean moments across the R simulations,

$$\bar{\Psi}_{R,T}^M(\hat{\theta}_i, \mathcal{E}^s) = \tfrac{1}{R}\textstyle\sum_{r=1}^R \Psi_T^M(\hat{\theta}_i, \mathcal{E}^s(r)),$$

and evaluate the loss function:

$$J_i = [\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\hat{\theta}_i, \mathcal{E}^s)]'[\hat{\Sigma}_T^D(1+1/R)]^{-1}[\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\hat{\theta}_i, \mathcal{E}^s)].$$

(b) Find a guess, $\hat{\theta}_0$, for the N_p estimated parameters and the covariance matrix, Σ_0:

i. Find the parameter draw $\hat{\theta}_0$ that corresponds to $\min\{J_i\}_{i=1}^{N_m}$.

ii. Find all J_i in the top decile, stack the corresponding draws in a $N_m/10 \times N_p$ matrix, $\hat{\Theta}$, and define the (i,j) element as $\tilde{\Theta}_{i,j} = \hat{\Theta}_{i,j} - \sum_{i=1}^{N_m/10} \hat{\Theta}_{i,j}/(N_m/10)$.

iii. Calculate $\Sigma_0 = \tilde{\Theta}'\tilde{\Theta}/(N_m/10)$.

(c) Minimize J with simulated annealing. For $i \in \{0, \ldots, N_d\}$, repeat the following steps:

i. Draw a candidate vector of parameters, $\hat{\theta}_i^{cand}$, where

$$\hat{\theta}_i^{cand} \sim \begin{cases} \hat{\theta}_0 & \text{for } i = 0, \\ \mathbb{N}(\hat{\theta}_{i-1}, c_0\Sigma_0) & \text{for } i > 0. \end{cases}$$

We set c_0 to target an acceptance rate of 50%. For the revised preferences, we restrict $\hat{\theta}_i^{cand}$ so that $\beta \exp(4(1-\beta)\sqrt{\sigma_a^2/(1-\rho_a^2)}) < 1$. This ensures the utility function weights are positive in 99.997% of the simulated observations.

ii. Repeat steps 2a, ii-iv.

iii. Accept or reject the candidate draw according to

$$(\hat{\theta}_i, J_i) = \begin{cases} (\hat{\theta}_i^{cand}, J_i^{cand}) & \text{if } i = 0, \\ (\hat{\theta}_i^{cand}, J_i^{cand}) & \text{if } \min(1, \exp(J_{i-1} - J_i^{cand})/c_1) > \hat{u}, \\ (\hat{\theta}_{i-1}, J_{i-1}) & \text{otherwise}, \end{cases}$$

where c_1 is the temperature and $\hat{u}$ is a draw from a uniform distribution.

(d) Find $\hat{\theta}_0^{up}$ and Σ_0^{up} following step 2b.

(e) Repeat steps 2c-d N_{SMM} times, initializing at draw $\hat{\theta}_0 = \hat{\theta}_0^{up}$ and covariance matrix $\Sigma_0 = \Sigma_0^{up}$. Gradually decrease the temperature. Of all the draws, find the lowest N_J J values, denoted $\{J_j^{guess}\}_{j=1}^{N_J}$, and the corresponding draws, $\{\theta_j^{guess}\}_{j=1}^{N_J}$.

(f) For $j \in \{1, \ldots, N_J\}$, minimize the same loss function with MATLAB's `fminsearch` starting at θ_j^{guess}. The resulting minimum is $\hat{\theta}_j^{\min}$ with a loss function value of $J_j^{\min}$. Repeat, each time updating the guess, until $J_j^{guess} - J_j^{\min} < 0.001$. The parameter estimates reported in the tables in the main paper, denoted $\hat{\theta}^s$, correspond to $\min\{J_j^{\min}\}_{j=1}^{N_J}$.

The set of SMM parameter estimates $\{\hat{\theta}^s\}_{s=1}^{N_s}$ approximate the joint sampling distribution of the parameters. We report its mean, $\bar{\theta} = \sum_{s=1}^{N_s} \hat{\theta}^s / N_s$, and $(5, 95)$ percentiles.

For all model specifications, the results in the paper are based on $N_s = 500$, $R = 1{,}000$, $N_{SMM} = 5$, $N_m = 10{,}000$, $N_d = 20{,}000$, and $N_J = 50$. The algorithm was programmed in Fortran and executed with Open MPI on the BigTex supercomputer at the Federal Reserve Bank of Dallas.

7 Estimation Robustness

Baseline Model: $\psi = 2.0$

	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Ptr	Current	Revised	Max RA	Current	Revised	Max RA
γ	1.48 (1.46, 1.50)	77.10 (75.41, 78.66)	10.00 (10.00, 10.00)	1.34 (1.32, 1.36)	99.17 (97.70, 100.61)	10.00 (10.00, 10.00)
β	0.9979 (0.9978, 0.9980)	0.9957 (0.9956, 0.9957)	0.9974 (0.9974, 0.9975)	0.9981 (0.9980, 0.9982)	0.9964 (0.9963, 0.9964)	0.9979 (0.9979, 0.9980)
ρ_a	0.9969 (0.9968, 0.9970)	0.9902 (0.9901, 0.9904)	0.9881 (0.9879, 0.9882)	0.9974 (0.9973, 0.9975)	0.9896 (0.9895, 0.9897)	0.9880 (0.9879, 0.9881)
σ_a	0.00030 (0.00029, 0.00030)	0.03491 (0.03469, 0.03511)	0.03856 (0.03836, 0.03875)	0.00027 (0.00026, 0.00027)	0.03597 (0.03579, 0.03615)	0.03869 (0.03848, 0.03888)
μ_y	0.0016 (0.0016, 0.0016)	0.0016 (0.0016, 0.0016)	0.0017 (0.0017, 0.0017)	0.0016 (0.0016, 0.0016)	0.0017 (0.0016, 0.0017)	0.0016 (0.0016, 0.0016)
μ_d	0.0015 (0.0015, 0.0015)	0.0021 (0.0020, 0.0021)	0.0010 (0.0009, 0.0010)	0.0010 (0.0010, 0.0010)	0.0017 (0.0016, 0.0017)	0.0005 (0.0004, 0.0005)
σ_y	0.0058 (0.0057, 0.0058)	0.0057 (0.0057, 0.0058)	0.0058 (0.0058, 0.0059)	0.0058 (0.0058, 0.0058)	0.0055 (0.0055, 0.0056)	0.0060 (0.0060, 0.0061)
ψ_d	1.51 (1.47, 1.55)	0.97 (0.93, 1.00)	1.07 (1.03, 1.10)	1.49 (1.45, 1.53)	1.12 (1.09, 1.15)	1.01 (0.98, 1.05)
π_{dy}	0.811 (0.785, 0.840)	0.434 (0.419, 0.450)	0.612 (0.589, 0.635)	0.809 (0.783, 0.838)	0.608 (0.597, 0.619)	0.601 (0.579, 0.625)
J	28.63 (28.03, 29.30)	47.74 (47.48, 48.02)	56.05 (55.62, 56.47)	30.81 (30.22, 31.46)	49.72 (49.42, 50.04)	59.78 (59.45, 60.13)
pval	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)
df	6	6	6	8	8	8

Table 7.1: Baseline model. Average and $(5, 95)$ percentiles of the parameter estimates.

		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Moment	Data	Current	Revised	Max RA	Current	Revised	Max RA
$E[\Delta c]$	1.89	1.89 (0.00)	1.94 (0.21)	2.00 (0.46)	1.89 (0.00)	1.98 (0.37)	1.95 (0.23)
$E[\Delta d]$	1.47	1.79 (0.33)	2.48 (1.05)	1.16 (-0.33)	1.22 (-0.26)	1.99 (0.54)	0.58 (-0.94)
$E[z_d]$	3.42	3.45 (0.24)	3.49 (0.49)	3.56 (1.04)	3.49 (0.49)	3.53 (0.75)	3.60 (1.29)
$E[r_d]$	6.51	5.60 (-0.57)	5.61 (-0.57)	4.04 (-1.55)	5.05 (-0.92)	5.00 (-0.94)	3.36 (-1.97)
$E[r_f]$	0.25	0.25 (0.00)	0.38 (0.20)	1.10 (1.39)	0.12 (-0.22)	0.27 (0.02)	0.46 (0.33)
$E[r_{f,5}]$	1.19	1.21 (0.03)	1.74 (0.80)	2.19 (1.46)	0.91 (-0.41)	1.22 (0.04)	1.52 (0.48)
$E[r_{f,20}]$	1.88	3.10 (2.04)	3.47 (2.65)	3.31 (2.39)	2.53 (1.08)	2.30 (0.70)	2.62 (1.24)
$SD[\Delta c]$	1.99	1.99 (0.00)	1.97 (-0.04)	2.00 (0.02)	2.00 (0.01)	1.91 (-0.17)	2.07 (0.17)
$SD[\Delta d]$	11.09	3.42 (-2.80)	2.09 (-3.29)	2.46 (-3.15)	3.39 (-2.82)	2.43 (-3.16)	2.44 (-3.16)
$SD[r_d]$	19.15	17.96 (-0.63)	13.48 (-2.99)	13.24 (-3.12)	17.99 (-0.61)	13.31 (-3.08)	12.92 (-3.28)
$SD[r_f]$	2.72	3.25 (1.04)	3.68 (1.89)	3.86 (2.24)	3.04 (0.62)	3.68 (1.89)	3.75 (2.03)
$SD[z_d]$	0.45	0.48 (0.44)	0.26 (-3.09)	0.23 (-3.46)	0.50 (0.73)	0.25 (-3.24)	0.23 (-3.56)
$AC[r_f]$	0.68	0.94 (4.00)	0.89 (3.28)	0.88 (3.04)	0.94 (4.05)	0.89 (3.21)	0.88 (3.03)
$AC[z_d]$	0.89	0.91 (0.42)	0.84 (-1.00)	0.82 (-1.48)	0.91 (0.52)	0.84 (-1.14)	0.82 (-1.50)
$Corr[\Delta c, \Delta d]$	0.54	0.47 (-0.32)	0.41 (-0.62)	0.50 (-0.20)	0.48 (-0.30)	0.48 (-0.29)	0.51 (-0.14)
$Corr[\Delta c, r_d]$	0.05	0.09 (0.58)	0.06 (0.22)	0.09 (0.62)	0.09 (0.58)	0.09 (0.54)	0.09 (0.67)
$Corr[\Delta d, r_d]$	0.07	0.19 (1.42)	0.15 (1.01)	0.18 (1.38)	0.18 (1.39)	0.18 (1.34)	0.19 (1.41)

Table 7.2: Baseline model. Data and average model-implied moments. t-statistics are in parentheses.

Baseline Model: $\psi = 1.5$

	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Ptr	Current	Revised	Max RA	Current	Revised	Max RA
γ	1.32 (1.31, 1.33)	80.19 (77.97, 81.90)	10.00 (10.00, 10.00)	1.22 (1.21, 1.24)	100.48 (98.96, 101.91)	10.00 (10.00, 10.00)
β	0.9982 (0.9981, 0.9982)	0.9957 (0.9957, 0.9958)	0.9977 (0.9976, 0.9977)	0.9983 (0.9982, 0.9984)	0.9964 (0.9964, 0.9964)	0.9982 (0.9981, 0.9982)
ρ_a	0.9969 (0.9968, 0.9970)	0.9902 (0.9901, 0.9903)	0.9878 (0.9876, 0.9879)	0.9974 (0.9973, 0.9975)	0.9896 (0.9895, 0.9897)	0.9877 (0.9876, 0.9878)
σ_a	0.00030 (0.00029, 0.00030)	0.03504 (0.03484, 0.03524)	0.03901 (0.03879, 0.03923)	0.00027 (0.00026, 0.00027)	0.03603 (0.03585, 0.03621)	0.03914 (0.03894, 0.03936)
μ_y	0.0016 (0.0016, 0.0016)	0.0016 (0.0016, 0.0016)	0.0017 (0.0016, 0.0017)	0.0016 (0.0016, 0.0016)	0.0017 (0.0016, 0.0017)	0.0016 (0.0016, 0.0016)
μ_d	0.0015 (0.0015, 0.0015)	0.0021 (0.0020, 0.0021)	0.0009 (0.0009, 0.0010)	0.0010 (0.0010, 0.0010)	0.0017 (0.0016, 0.0017)	0.0005 (0.0004, 0.0005)
σ_y	0.0058 (0.0057, 0.0058)	0.0057 (0.0056, 0.0057)	0.0058 (0.0058, 0.0059)	0.0058 (0.0058, 0.0058)	0.0055 (0.0055, 0.0055)	0.0060 (0.0060, 0.0061)
ψ_d	1.51 (1.47, 1.55)	0.98 (0.94, 1.01)	1.05 (1.02, 1.09)	1.49 (1.45, 1.53)	1.12 (1.10, 1.15)	1.00 (0.96, 1.03)
π_{dy}	0.811 (0.785, 0.840)	0.439 (0.422, 0.456)	0.605 (0.583, 0.629)	0.809 (0.782, 0.838)	0.611 (0.599, 0.622)	0.595 (0.573, 0.619)
J	28.63 (28.03, 29.30)	47.93 (47.67, 48.21)	57.01 (56.57, 57.43)	30.82 (30.23, 31.47)	49.80 (49.50, 50.12)	60.71 (60.37, 61.06)
pval	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)	0.000 (0.000, 0.000)
df	6	6	6	8	8	8

Table 7.3: Baseline model. Average and $(5, 95)$ percentiles of the parameter estimates.

		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Moment	Data	Current	Revised	Max RA	Current	Revised	Max RA
$E[\Delta c]$	1.89	1.89 (0.00)	1.95 (0.23)	1.99 (0.40)	1.89 (0.00)	1.98 (0.38)	1.93 (0.18)
$E[\Delta d]$	1.47	1.79 (0.33)	2.46 (1.03)	1.13 (−0.36)	1.22 (−0.26)	1.99 (0.54)	0.54 (−0.97)
$E[z_d]$	3.42	3.45 (0.24)	3.49 (0.50)	3.56 (1.04)	3.49 (0.49)	3.52 (0.75)	3.60 (1.28)
$E[r_d]$	6.51	5.60 (−0.57)	5.59 (−0.58)	4.01 (−1.57)	5.05 (−0.92)	5.00 (−0.95)	3.33 (−1.99)
$E[r_f]$	0.25	0.25 (0.00)	0.39 (0.22)	1.12 (1.42)	0.12 (−0.22)	0.28 (0.03)	0.47 (0.36)
$E[r_{f,5}]$	1.19	1.21 (0.03)	1.73 (0.79)	2.20 (1.49)	0.91 (−0.41)	1.23 (0.05)	1.53 (0.49)
$E[r_{f,20}]$	1.88	3.11 (2.05)	3.43 (2.58)	3.30 (2.37)	2.53 (1.09)	2.29 (0.68)	2.61 (1.22)
$SD[\Delta c]$	1.99	1.99 (0.00)	1.95 (−0.08)	2.02 (0.05)	2.00 (0.01)	1.90 (−0.19)	2.08 (0.19)
$SD[\Delta d]$	11.09	3.42 (−2.80)	2.09 (−3.29)	2.44 (−3.16)	3.38 (−2.82)	2.43 (−3.17)	2.42 (−3.17)
$SD[r_d]$	19.15	17.96 (−0.63)	13.46 (−3.00)	13.15 (−3.16)	17.99 (−0.61)	13.30 (−3.08)	12.85 (−3.32)
$SD[r_f]$	2.72	3.25 (1.04)	3.68 (1.90)	3.87 (2.27)	3.03 (0.62)	3.68 (1.90)	3.77 (2.06)
$SD[z_d]$	0.45	0.48 (0.44)	0.25 (−3.10)	0.23 (−3.52)	0.50 (0.73)	0.25 (−3.25)	0.22 (−3.62)
$AC[r_f]$	0.68	0.94 (4.00)	0.89 (3.27)	0.88 (3.01)	0.94 (4.05)	0.89 (3.20)	0.87 (3.00)
$AC[z_d]$	0.89	0.91 (0.42)	0.84 (−1.01)	0.82 (−1.54)	0.91 (0.52)	0.83 (−1.15)	0.82 (−1.56)
$Corr[\Delta c, \Delta d]$	0.54	0.47 (−0.32)	0.41 (−0.62)	0.50 (−0.19)	0.47 (−0.30)	0.48 (−0.29)	0.51 (−0.13)
$Corr[\Delta c, r_d]$	0.05	0.09 (0.58)	0.06 (0.22)	0.09 (0.62)	0.09 (0.57)	0.09 (0.54)	0.09 (0.67)
$Corr[\Delta d, r_d]$	0.07	0.19 (1.42)	0.15 (1.01)	0.18 (1.37)	0.18 (1.39)	0.18 (1.34)	0.19 (1.41)

Table 7.4: Baseline model. Data and average model-implied moments. t-statistics are in parentheses.

Long-Run Risk Model: $\psi = 2.0$

	Omits $SD[r_f]$, $AC[r_f]$, $E[r_{f,5}]$, & $E[r_{f,20}]$		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$		All Moments	
Parameter	No VR	Revised	No VR	Revised	No VR	Revised
γ	2.52 $(2.45, 2.60)$	2.48 $(2.36, 2.63)$	2.59 $(2.47, 2.70)$	2.52 $(2.43, 2.61)$	2.39 $(2.04, 2.64)$	2.30 $(2.21, 2.40)$
β	0.9992 $(0.9992, 0.9993)$	0.9983 $(0.9982, 0.9984)$	0.9992 $(0.9992, 0.9993)$	0.9992 $(0.9992, 0.9992)$	0.9987 $(0.9986, 0.9988)$	0.9987 $(0.9987, 0.9988)$
ρ_a	–	0.9808 $(0.9800, 0.9815)$	–	0.9553 $(0.9544, 0.9561)$	–	0.9579 $(0.9569, 0.9588)$
σ_a	–	0.0488 $(0.0479, 0.0498)$	–	0.0167 $(0.0165, 0.0170)$	–	0.0177 $(0.0174, 0.0181)$
μ_y	0.0016 $(0.0015, 0.0016)$	0.0016 $(0.0014, 0.0017)$	0.0016 $(0.0015, 0.0017)$	0.0016 $(0.0015, 0.0017)$	0.0016 $(0.0015, 0.0017)$	0.0016 $(0.0015, 0.0017)$
μ_d	0.0012 $(0.0010, 0.0014)$	0.0011 $(0.0009, 0.0014)$	0.0014 $(0.0012, 0.0016)$	0.0012 $(0.0009, 0.0014)$	0.0011 $(0.0009, 0.0013)$	0.0010 $(0.0007, 0.0012)$
σ_y	0.0040 $(0.0039, 0.0040)$	0.0039 $(0.0038, 0.0040)$	0.0048 $(0.0048, 0.0048)$	0.0039 $(0.0039, 0.0039)$	0.0045 $(0.0044, 0.0045)$	0.0036 $(0.0035, 0.0036)$
ψ_d	3.36 $(3.28, 3.43)$	2.85 $(2.74, 2.99)$	3.11 $(3.03, 3.16)$	3.35 $(3.27, 3.43)$	3.28 $(3.08, 3.42)$	3.54 $(3.45, 3.62)$
π_{dy}	0.632 $(0.548, 0.712)$	0.896 $(0.816, 0.979)$	0.071 $(-0.071, 0.198)$	0.707 $(0.631, 0.777)$	0.174 $(-0.031, 0.416)$	0.863 $(0.786, 0.933)$
ϕ_d	2.36 $(2.27, 2.44)$	1.54 $(1.47, 1.61)$	2.16 $(2.06, 2.25)$	2.31 $(2.23, 2.40)$	2.37 $(2.10, 2.56)$	2.46 $(2.38, 2.55)$
ρ_x	0.9988 $(0.9987, 0.9989)$	0.9995 $(0.9995, 0.9995)$	0.9979 $(0.9976, 0.9982)$	0.9989 $(0.9988, 0.9990)$	0.9978 $(0.9972, 0.9986)$	0.9990 $(0.9989, 0.9991)$
ψ_x	0.0262 $(0.0255, 0.0268)$	0.0265 $(0.0257, 0.0274)$	0.0317 $(0.0308, 0.0326)$	0.0258 $(0.0252, 0.0264)$	0.0304 $(0.0288, 0.0316)$	0.0248 $(0.0242, 0.0255)$
J	21.10 $(20.45, 21.81)$	13.39 $(13.16, 13.63)$	53.02 $(52.23, 53.96)$	19.90 $(19.34, 20.46)$	60.64 $(59.95, 61.45)$	25.19 $(24.64, 25.72)$
pval	0.007 $(0.005, 0.009)$	0.037 $(0.034, 0.041)$	0.000 $(0.000, 0.000)$	0.011 $(0.009, 0.013)$	0.000 $(0.000, 0.000)$	0.005 $(0.004, 0.006)$
df	8	6	10	8	12	10

Table 7.5: Long-run risk model. Average and $(5, 95)$ percentiles of the parameter estimates.

Moment	Data	Omits $SD[r_f]$, $AC[r_f]$, $E[r_{f,5}]$, & $E[r_{f,20}]$ No VR	Omits $SD[r_f]$, $AC[r_f]$, $E[r_{f,5}]$, & $E[r_{f,20}]$ Revised	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$ No VR	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$ Revised	All Moments No VR	All Moments Revised
$E[\Delta c]$	1.89	1.87 (−0.09)	1.89 (0.01)	1.87 (−0.05)	1.89 (0.00)	1.89 (0.00)	1.89 (0.01)
$E[\Delta d]$	1.47	1.45 (−0.03)	1.36 (−0.12)	1.65 (0.18)	1.40 (−0.08)	1.36 (−0.12)	1.18 (−0.31)
$E[z_d]$	3.42	3.42 (0.00)	3.42 (−0.03)	3.41 (−0.06)	3.42 (0.02)	3.43 (0.05)	3.44 (0.10)
$E[r_d]$	6.51	6.69 (0.11)	6.94 (0.27)	6.08 (−0.27)	6.71 (0.12)	5.76 (−0.47)	6.62 (0.07)
$E[r_f]$	0.25	0.36 (0.18)	0.27 (0.03)	0.32 (0.10)	0.26 (0.01)	1.46 (1.98)	1.24 (1.62)
$E[r_{f,5}]$	1.19	0.18 (−1.50)	1.01 (−0.27)	0.05 (−1.69)	0.22 (−1.44)	1.27 (0.12)	1.27 (0.12)
$E[r_{f,20}]$	1.88	−0.34 (−3.69)	0.86 (−1.68)	−0.68 (−4.25)	−0.27 (−3.56)	0.77 (−1.83)	0.95 (−1.54)
$SD[\Delta c]$	1.99	1.95 (−0.09)	2.01 (0.03)	2.47 (0.99)	1.90 (−0.19)	2.25 (0.53)	1.73 (−0.55)
$SD[\Delta d]$	11.09	5.74 (−1.96)	4.67 (−2.35)	6.52 (−1.67)	5.57 (−2.02)	6.40 (−1.71)	5.41 (−2.08)
$SD[r_d]$	19.15	17.53 (−0.85)	19.38 (0.13)	18.41 (−0.39)	17.65 (−0.79)	18.50 (−0.34)	17.65 (−0.79)
$SD[r_f]$	2.72	0.67 (−4.04)	5.73 (5.94)	0.90 (−3.60)	2.84 (0.24)	0.80 (−3.79)	2.94 (0.43)
$SD[z_d]$	0.45	0.55 (1.61)	0.46 (0.17)	0.53 (1.34)	0.54 (1.44)	0.54 (1.36)	0.54 (1.46)
$AC[\Delta c]$	0.53	0.43 (−1.05)	0.47 (−0.68)	0.48 (−0.54)	0.43 (−1.08)	0.46 (−0.77)	0.42 (−1.21)
$AC[\Delta d]$	0.19	0.28 (0.86)	0.21 (0.17)	0.33 (1.27)	0.27 (0.76)	0.32 (1.22)	0.26 (0.67)
$AC[r_d]$	−0.01	0.01 (0.21)	−0.05 (−0.47)	0.00 (0.11)	−0.01 (0.03)	0.00 (0.10)	−0.01 (0.04)
$AC[r_f]$	0.68	0.95 (4.20)	0.83 (2.30)	0.95 (4.10)	0.69 (0.16)	0.95 (4.10)	0.70 (0.31)
$AC[z_d]$	0.89	0.93 (0.80)	0.88 (−0.12)	0.92 (0.61)	0.92 (0.72)	0.92 (0.61)	0.92 (0.72)
$Corr[\Delta c, \Delta d]$	0.54	0.49 (−0.24)	0.52 (−0.09)	0.44 (−0.48)	0.49 (−0.21)	0.44 (−0.45)	0.50 (−0.16)
$Corr[\Delta c, r_d]$	0.05	0.07 (0.27)	0.06 (0.13)	0.08 (0.49)	0.06 (0.24)	0.08 (0.49)	0.06 (0.21)
$Corr[\Delta d, r_d]$	0.07	0.24 (2.08)	0.19 (1.42)	0.28 (2.57)	0.23 (1.97)	0.28 (2.49)	0.22 (1.86)
$Corr[ep, z_{d,-1}]$	−0.16	−0.18 (−0.12)	−0.13 (0.32)	−0.14 (0.25)	−0.17 (−0.08)	−0.14 (0.24)	−0.18 (−0.12)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.65 (2.63)	0.58 (2.21)	0.69 (2.83)	0.65 (2.59)	0.67 (2.74)	0.64 (2.54)

Table 7.6: Long-run risk model. Data and average model-implied moments. t-statistics are in parentheses.

Long-Run Risk Model: $\psi = 1.5$

	Omits $SD[r_f]$, $AC[r_f]$, $E[r_{f,5}]$, & $E[r_{f,20}]$		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$		All Moments	
Parameter	No VR	Revised	No VR	Revised	No VR	Revised
γ	2.21 $(2.15, 2.27)$	2.36 $(2.25, 2.50)$	2.09 $(2.01, 2.17)$	2.22 $(2.16, 2.28)$	1.93 $(1.75, 2.37)$	2.18 $(2.10, 2.26)$
β	0.9995 $(0.9995, 0.9995)$	0.9988 $(0.9987, 0.9989)$	0.9995 $(0.9994, 0.9995)$	0.9995 $(0.9995, 0.9995)$	0.9991 $(0.9990, 0.9992)$	0.9992 $(0.9992, 0.9992)$
ρ_a	–	0.9798 $(0.9789, 0.9808)$	–	0.9534 $(0.9524, 0.9544)$	–	0.9568 $(0.9558, 0.9578)$
σ_a	–	0.0500 $(0.0488, 0.0511)$	–	0.0164 $(0.0161, 0.0167)$	–	0.0176 $(0.0173, 0.0179)$
μ_y	0.0015 $(0.0014, 0.0016)$	0.0016 $(0.0014, 0.0017)$	0.0015 $(0.0014, 0.0017)$	0.0015 $(0.0014, 0.0016)$	0.0016 $(0.0015, 0.0017)$	0.0016 $(0.0015, 0.0017)$
μ_d	0.0011 $(0.0009, 0.0014)$	0.0011 $(0.0009, 0.0014)$	0.0012 $(0.0009, 0.0015)$	0.0011 $(0.0009, 0.0014)$	0.0009 $(0.0006, 0.0012)$	0.0009 $(0.0006, 0.0011)$
σ_y	0.0041 $(0.0040, 0.0041)$	0.0039 $(0.0037, 0.0040)$	0.0049 $(0.0048, 0.0049)$	0.0039 $(0.0039, 0.0040)$	0.0045 $(0.0044, 0.0045)$	0.0034 $(0.0033, 0.0034)$
ψ_d	3.34 $(3.27, 3.40)$	3.07 $(2.93, 3.25)$	3.00 $(2.95, 3.06)$	3.36 $(3.29, 3.43)$	3.19 $(3.08, 3.44)$	3.80 $(3.71, 3.89)$
π_{dy}	0.637 $(0.561, 0.720)$	0.854 $(0.763, 0.968)$	0.264 $(0.155, 0.368)$	0.707 $(0.629, 0.779)$	0.394 $(0.059, 0.546)$	0.913 $(0.832, 0.988)$
ϕ_d	2.35 $(2.27, 2.42)$	1.73 $(1.67, 1.81)$	2.01 $(1.94, 2.08)$	2.35 $(2.27, 2.42)$	2.25 $(2.10, 2.62)$	2.72 $(2.62, 2.81)$
ρ_x	0.9991 $(0.9990, 0.9991)$	0.9995 $(0.9995, 0.9995)$	0.9988 $(0.9987, 0.9990)$	0.9991 $(0.9991, 0.9992)$	0.9987 $(0.9978, 0.9991)$	0.9991 $(0.9990, 0.9992)$
ψ_x	0.0258 $(0.0251, 0.0264)$	0.0264 $(0.0255, 0.0273)$	0.0295 $(0.0287, 0.0302)$	0.0253 $(0.0247, 0.0260)$	0.0280 $(0.0270, 0.0298)$	0.0243 $(0.0237, 0.0249)$
J	22.22 $(21.54, 22.96)$	13.58 $(13.32, 13.86)$	50.44 $(49.55, 51.38)$	20.73 $(20.14, 21.34)$	59.22 $(58.44, 60.05)$	26.48 $(25.92, 27.02)$
pval	0.005 $(0.003, 0.006)$	0.035 $(0.031, 0.038)$	0.000 $(0.000, 0.000)$	0.008 $(0.006, 0.010)$	0.000 $(0.000, 0.000)$	0.003 $(0.003, 0.004)$
df	8	6	10	8	12	10

Table 7.7: Long-run risk model. Average and $(5, 95)$ percentiles of the parameter estimates.

		Omits $SD[r_f]$, $AC[r_f]$, $E[r_{f,5}]$, & $E[r_{f,20}]$		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$		All Moments	
Moment	Data	No VR	Revised	No VR	Revised	No VR	Revised
$E[\Delta c]$	1.89	1.82 (−0.25)	1.89 (0.01)	1.85 (−0.13)	1.84 (−0.17)	1.89 (0.00)	1.89 (0.01)
$E[\Delta d]$	1.47	1.39 (−0.09)	1.36 (−0.12)	1.46 (−0.01)	1.35 (−0.13)	1.04 (−0.45)	1.07 (−0.42)
$E[z_d]$	3.42	3.43 (0.08)	3.42 (−0.02)	3.43 (0.03)	3.43 (0.08)	3.45 (0.18)	3.44 (0.16)
$E[r_d]$	6.51	7.10 (0.37)	6.97 (0.29)	6.74 (0.14)	7.09 (0.36)	6.22 (−0.18)	6.71 (0.12)
$E[r_f]$	0.25	0.64 (0.63)	0.28 (0.03)	0.44 (0.31)	0.51 (0.42)	1.54 (2.12)	1.29 (1.71)
$E[r_{f,5}]$	1.19	0.41 (−1.16)	0.96 (−0.34)	0.11 (−1.60)	0.42 (−1.15)	1.33 (0.21)	1.30 (0.16)
$E[r_{f,20}]$	1.88	−0.29 (−3.59)	0.62 (−2.09)	−0.88 (−4.58)	−0.23 (−3.50)	0.71 (−1.94)	0.90 (−1.62)
$SD[\Delta c]$	1.99	2.00 (0.01)	1.97 (−0.04)	2.54 (1.12)	1.93 (−0.14)	2.25 (0.53)	1.62 (−0.77)
$SD[\Delta d]$	11.09	5.85 (−1.92)	4.96 (−2.24)	6.37 (−1.72)	5.69 (−1.98)	6.20 (−1.79)	5.51 (−2.04)
$SD[r_d]$	19.15	17.27 (−0.99)	19.47 (0.17)	17.69 (−0.77)	17.41 (−0.92)	17.87 (−0.67)	17.59 (−0.82)
$SD[r_f]$	2.72	0.92 (−3.55)	5.89 (6.25)	1.24 (−2.93)	2.89 (0.32)	1.06 (−3.27)	2.95 (0.46)
$SD[z_d]$	0.45	0.55 (1.64)	0.46 (0.19)	0.56 (1.67)	0.54 (1.46)	0.56 (1.70)	0.54 (1.49)
$AC[\Delta c]$	0.53	0.44 (−1.02)	0.47 (−0.70)	0.49 (−0.48)	0.43 (−1.06)	0.46 (−0.77)	0.41 (−1.28)
$AC[\Delta d]$	0.19	0.29 (0.90)	0.23 (0.32)	0.32 (1.17)	0.28 (0.81)	0.31 (1.10)	0.27 (0.72)
$AC[r_d]$	−0.01	0.02 (0.31)	−0.05 (−0.49)	0.01 (0.25)	0.00 (0.12)	0.01 (0.24)	0.00 (0.08)
$AC[r_f]$	0.68	0.95 (4.22)	0.83 (2.23)	0.95 (4.20)	0.70 (0.21)	0.95 (4.19)	0.70 (0.31)
$AC[z_d]$	0.89	0.93 (0.85)	0.88 (−0.14)	0.93 (0.80)	0.93 (0.76)	0.93 (0.78)	0.93 (0.74)
$Corr[\Delta c, \Delta d]$	0.54	0.49 (−0.22)	0.51 (−0.13)	0.47 (−0.34)	0.50 (−0.20)	0.47 (−0.31)	0.50 (−0.17)
$Corr[\Delta c, r_d]$	0.05	0.06 (0.24)	0.06 (0.19)	0.07 (0.34)	0.06 (0.22)	0.07 (0.35)	0.06 (0.21)
$Corr[\Delta d, r_d]$	0.07	0.24 (2.12)	0.20 (1.56)	0.27 (2.41)	0.24 (2.02)	0.26 (2.30)	0.23 (1.90)
$Corr[ep, z_{d,-1}]$	−0.16	−0.19 (−0.28)	−0.13 (0.32)	−0.18 (−0.13)	−0.18 (−0.21)	−0.17 (−0.11)	−0.18 (−0.18)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.65 (2.64)	0.58 (2.22)	0.69 (2.85)	0.65 (2.60)	0.67 (2.74)	0.63 (2.51)

Table 7.8: Long-run risk model. Data and average model-implied moments. t-statistics are in parentheses.

Extended Long-Run Risk Model: $\psi = 2.0$

	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Ptr	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
γ	2.51 (2.46, 2.56)	3.21 (3.09, 3.33)	5.90 (5.52, 6.36)	1.27 (1.08, 1.50)	3.62 (3.48, 3.77)	8.05 (7.66, 8.46)
β	0.9986 (0.9985, 0.9986)	0.9992 (0.9992, 0.9993)	0.9984 (0.9983, 0.9985)	0.9984 (0.9983, 0.9986)	0.9990 (0.9990, 0.9990)	0.9979 (0.9978, 0.9979)
ρ_a	–	0.9602 (0.9593, 0.9610)	0.9932 (0.9931, 0.9934)	–	0.9629 (0.9620, 0.9636)	0.9936 (0.9934, 0.9937)
σ_a	–	0.0187 (0.0184, 0.0190)	0.0291 (0.0287, 0.0294)	–	0.0198 (0.0194, 0.0201)	0.0283 (0.0280, 0.0286)
μ_y	0.0016 (0.0015, 0.0017)	0.0015 (0.0015, 0.0016)	0.0016 (0.0015, 0.0016)	0.0017 (0.0015, 0.0018)	0.0016 (0.0015, 0.0016)	0.0016 (0.0015, 0.0016)
μ_d	0.0012 (0.0010, 0.0015)	0.0014 (0.0013, 0.0016)	0.0015 (0.0014, 0.0017)	0.0000 (0.0000, 0.0000)	0.0013 (0.0011, 0.0015)	0.0014 (0.0013, 0.0016)
σ_y	0.0003 (0.0002, 0.0003)	0.0040 (0.0040, 0.0040)	0.0000 (0.0000, 0.0001)	0.0002 (0.0000, 0.0004)	0.0035 (0.0034, 0.0035)	0.0000 (0.0000, 0.0000)
ψ_d	3.05 (2.98, 3.12)	–	–	2.83 (2.77, 2.89)	–	–
π_{dy}	0.785 (0.689, 0.876)	–	–	0.902 (0.804, 1.002)	–	–
ϕ_d	1.93 (1.88, 1.97)	2.76 (2.68, 2.84)	2.79 (2.73, 2.85)	1.73 (1.70, 1.77)	3.50 (3.40, 3.59)	2.82 (2.78, 2.86)
ρ_x	0.9991 (0.9990, 0.9992)	0.9975 (0.9973, 0.9977)	0.9961 (0.9959, 0.9963)	0.9995 (0.9995, 0.9995)	0.9968 (0.9966, 0.9969)	0.9961 (0.9959, 0.9963)
ψ_x	0.0267 (0.0260, 0.0274)	0.0301 (0.0295, 0.0307)	0.0361 (0.0352, 0.0371)	0.0260 (0.0253, 0.0267)	0.0303 (0.0297, 0.0309)	0.0356 (0.0347, 0.0364)
π_{ya}	–	-0.048 $(-0.052, -0.044)$	-0.049 $(-0.051, -0.048)$	–	-0.031 $(-0.034, -0.028)$	-0.044 $(-0.047, -0.042)$
π_{da}	–	-1.026 $(-1.040, -1.011)$	-0.846 $(-0.858, -0.835)$	–	-0.993 $(-1.007, -0.979)$	-0.879 $(-0.891, -0.867)$
ρ_{σ_y}	0.9624 (0.9610, 0.9637)	–	0.8196 (0.7804, 0.8478)	0.9572 (0.9530, 0.9613)	–	0.5154 (0.4549, 0.5656)
ν_y	1.2e−5 (1.2e−5, 1.3e−5)	–	2.3e−5 (2.2e−5, 2.5e−5)	1.4e−5 (1.3e−5, 1.5e−5)	–	3.7e−5 (3.5e−5, 3.9e−5)
J	18.73 (18.21, 19.27)	14.00 (13.58, 14.48)	9.35 (9.06, 9.64)	26.37 (25.56, 27.26)	19.10 (18.71, 19.49)	10.88 (10.51, 11.26)
pval	0.016 (0.014, 0.020)	0.082 (0.070, 0.093)	0.155 (0.141, 0.170)	0.003 (0.002, 0.004)	0.039 (0.034, 0.044)	0.209 (0.188, 0.231)
df	8	8	6	10	10	8

Table 7.9: Extended long-run risk models. Average and $(5, 95)$ percentiles of the parameter estimates.

		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Moment	Data	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
$E[\Delta c]$	1.89	1.91 (0.10)	1.83 (-0.22)	1.89 (0.02)	1.99 (0.40)	1.89 (0.02)	1.92 (0.14)
$E[\Delta d]$	1.47	1.48 (0.00)	1.74 (0.28)	1.84 (0.38)	0.01 (-1.53)	1.55 (0.08)	1.74 (0.28)
$E[z_d]$	3.42	3.42 (0.00)	3.40 (-0.14)	3.40 (-0.19)	3.53 (0.79)	3.42 (-0.04)	3.40 (-0.15)
$E[r_d]$	6.51	6.96 (0.28)	6.01 (-0.32)	5.82 (-0.43)	6.60 (0.05)	5.58 (-0.59)	5.74 (-0.48)
$E[r_f]$	0.25	0.05 (-0.34)	0.54 (0.47)	0.16 (-0.16)	0.84 (0.96)	1.25 (1.64)	0.34 (0.14)
$E[r_{f,5}]$	1.19	-0.84 (-3.00)	0.46 (-1.08)	0.43 (-1.13)	1.45 (0.37)	1.27 (0.11)	1.46 (0.39)
$E[r_{f,20}]$	1.88	-2.44 (-7.18)	-0.06 (-3.22)	0.05 (-3.03)	1.30 (-0.96)	0.93 (-1.57)	1.47 (-0.68)
$SD[\Delta c]$	1.99	2.09 (0.20)	1.99 (0.00)	2.12 (0.27)	2.17 (0.37)	1.68 (-0.64)	2.15 (0.33)
$SD[\Delta d]$	11.09	5.43 (-2.07)	7.69 (-1.24)	9.52 (-0.57)	5.23 (-2.14)	7.92 (-1.16)	9.66 (-0.52)
$SD[r_d]$	19.15	18.14 (-0.53)	17.84 (-0.69)	18.31 (-0.44)	17.24 (-1.01)	18.35 (-0.42)	18.11 (-0.55)
$SD[r_f]$	2.72	2.38 (-0.67)	3.01 (0.57)	2.66 (-0.13)	2.58 (-0.27)	3.07 (0.70)	2.55 (-0.34)
$SD[z_d]$	0.45	0.53 (1.20)	0.52 (1.04)	0.49 (0.66)	0.56 (1.69)	0.50 (0.84)	0.51 (0.87)
$AC[\Delta c]$	0.53	0.45 (-0.90)	0.43 (-1.06)	0.45 (-0.88)	0.45 (-0.88)	0.41 (-1.27)	0.45 (-0.88)
$AC[\Delta d]$	0.19	0.25 (0.59)	0.22 (0.29)	0.17 (-0.18)	0.24 (0.47)	0.23 (0.36)	0.18 (-0.14)
$AC[r_d]$	-0.01	-0.03 (-0.29)	0.02 (0.35)	-0.03 (-0.30)	0.03 (0.46)	0.02 (0.32)	0.00 (0.07)
$AC[r_f]$	0.68	0.69 (0.09)	0.72 (0.53)	0.70 (0.35)	0.66 (-0.34)	0.73 (0.69)	0.72 (0.59)
$AC[z_d]$	0.89	0.91 (0.46)	0.92 (0.57)	0.90 (0.19)	0.93 (0.82)	0.91 (0.42)	0.91 (0.32)
$Corr[\Delta c, \Delta d]$	0.54	0.51 (-0.13)	0.47 (-0.30)	0.52 (-0.08)	0.54 (0.00)	0.45 (-0.43)	0.50 (-0.20)
$Corr[\Delta c, r_d]$	0.05	0.06 (0.20)	0.09 (0.65)	0.11 (0.82)	0.05 (0.03)	0.10 (0.72)	0.11 (0.85)
$Corr[\Delta d, r_d]$	0.07	0.22 (1.80)	0.14 (0.83)	0.07 (-0.01)	0.21 (1.69)	0.13 (0.79)	0.06 (-0.08)
$Corr[ep, z_{d,-1}]$	-0.16	-0.23 (-0.70)	-0.14 (0.30)	-0.13 (0.39)	-0.23 (-0.66)	-0.12 (0.45)	-0.11 (0.53)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.65 (2.60)	0.65 (2.62)	0.62 (2.45)	0.66 (2.67)	0.64 (2.54)	0.62 (2.46)

Table 7.10: Extended long-run risk models. Data and average model-implied moments. t-statistics are in parentheses.

Extended Long-Run Risk Model: $\psi = 1.5$

	Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Ptr	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
γ	2.23 (2.18, 2.27)	2.89 (2.73, 3.05)	4.56 (2.70, 5.03)	1.47 (1.16, 2.53)	3.67 (3.50, 3.84)	7.06 (5.78, 9.07)
β	0.9991 (0.9991, 0.9991)	0.9995 (0.9995, 0.9995)	0.9991 (0.9990, 0.9995)	0.9989 (0.9988, 0.9990)	0.9994 (0.9994, 0.9994)	0.9986 (0.9984, 0.9988)
ρ_a	–	0.9587 (0.9577, 0.9597)	0.9904 (0.9560, 0.9934)	–	0.9626 (0.9617, 0.9635)	0.9868 (0.9740, 0.9937)
σ_a	–	0.0184 (0.0180, 0.0187)	0.0280 (0.0170, 0.0293)	–	0.0199 (0.0195, 0.0203)	0.0244 (0.0202, 0.0274)
μ_y	0.0016 (0.0015, 0.0017)	0.0015 (0.0015, 0.0016)	0.0016 (0.0015, 0.0016)	0.0017 (0.0015, 0.0019)	0.0016 (0.0015, 0.0016)	0.0016 (0.0015, 0.0017)
μ_d	0.0011 (0.0009, 0.0014)	0.0014 (0.0012, 0.0016)	0.0015 (0.0013, 0.0017)	0.0001 (0.0000, 0.0007)	0.0012 (0.0011, 0.0014)	0.0013 (0.0010, 0.0015)
σ_y	0.0003 (0.0002, 0.0003)	0.0040 (0.0039, 0.0040)	0.0000 (0.0000, 0.0001)	0.0004 (0.0002, 0.0026)	0.0032 (0.0032, 0.0033)	0.0000 (0.0000, 0.0000)
ψ_d	3.10 (3.03, 3.17)	–	–	2.92 (2.85, 3.00)	–	–
π_{dy}	0.755 (0.661, 0.851)	–	–	0.782 (0.465, 0.905)	–	–
ϕ_d	1.99 (1.94, 2.05)	2.79 (2.69, 2.91)	2.60 (2.52, 2.67)	1.85 (1.79, 1.92)	3.88 (3.77, 3.98)	2.67 (2.45, 3.09)
ρ_x	0.9993 (0.9992, 0.9993)	0.9980 (0.9977, 0.9982)	0.9973 (0.9972, 0.9975)	0.9995 (0.9992, 0.9995)	0.9969 (0.9967, 0.9971)	0.9978 (0.9974, 0.9981)
ψ_x	0.0264 (0.0257, 0.0271)	0.0287 (0.0281, 0.0293)	0.0329 (0.0319, 0.0339)	0.0264 (0.0256, 0.0284)	0.0295 (0.0289, 0.0300)	0.0305 (0.0293, 0.0317)
π_{ya}	–	-0.045 $(-0.050, -0.039)$	-0.051 $(-0.053, -0.049)$	–	-0.028 $(-0.031, -0.026)$	-0.049 $(-0.054, -0.042)$
π_{da}	–	-1.037 $(-1.053, -1.022)$	-0.841 $(-1.066, -0.810)$	–	-0.986 $(-1.000, -0.972)$	-0.930 $(-1.008, -0.875)$
ρ_{σ_y}	0.9561 (0.9544, 0.9577)	–	0.8263 (0.7822, 0.9899)	0.8891 (0.0000, 0.9510)	–	0.2505 (0.0000, 0.4648)
ν_y	1.3e−5 (1.3e−5, 1.4e−5)	–	2.3e−5 (5.7e−6, 2.7e−5)	1.8e−5 (1.5e−5, 5.0e−5)	–	3.9e−5 (3.7e−5, 4.2e−5)
J	19.78 (19.23, 20.32)	15.43 (14.93, 15.97)	10.77 (10.40, 11.16)	29.22 (28.02, 31.89)	20.48 (20.06, 20.90)	14.36 (13.86, 14.81)
pval	0.011 (0.009, 0.014)	0.052 (0.043, 0.061)	0.096 (0.084, 0.109)	0.001 (0.000, 0.002)	0.025 (0.022, 0.029)	0.073 (0.063, 0.085)
df	8	8	6	10	10	8

Table 7.11: Extended long-run risk models. Average and $(5, 95)$ percentiles of the parameter estimates.

		Omits $E[r_{f,5}]$ & $E[r_{f,20}]$			All Moments		
Moment	Data	No VR+SV	Demand	Demand+SV	No VR+SV	Demand	Demand+SV
$E[\Delta c]$	1.89	1.93 (0.15)	1.81 (−0.32)	1.90 (0.05)	2.01 (0.48)	1.89 (0.02)	1.93 (0.18)
$E[\Delta d]$	1.47	1.37 (−0.11)	1.72 (0.25)	1.81 (0.36)	0.06 (−1.47)	1.50 (0.03)	1.53 (0.05)
$E[z_d]$	3.42	3.44 (0.09)	3.41 (−0.12)	3.40 (−0.18)	3.53 (0.81)	3.42 (−0.01)	3.41 (−0.07)
$E[r_d]$	6.51	7.21 (0.44)	6.19 (−0.20)	6.02 (−0.31)	6.55 (0.02)	5.54 (−0.61)	5.90 (−0.38)
$E[r_f]$	0.25	−0.01 (−0.44)	0.79 (0.87)	0.08 (−0.30)	0.94 (1.14)	1.30 (1.73)	0.34 (0.14)
$E[r_{f,5}]$	1.19	−0.86 (−3.04)	0.66 (−0.79)	0.10 (−1.62)	1.45 (0.38)	1.30 (0.15)	1.63 (0.64)
$E[r_{f,20}]$	1.88	−2.72 (−7.63)	−0.02 (−3.15)	−0.85 (−4.53)	1.10 (−1.29)	0.88 (−1.65)	1.28 (−0.99)
$SD[\Delta c]$	1.99	2.12 (0.26)	1.97 (−0.04)	2.15 (0.33)	2.25 (0.53)	1.56 (−0.90)	2.09 (0.19)
$SD[\Delta d]$	11.09	5.59 (−2.01)	7.66 (−1.25)	9.05 (−0.75)	5.58 (−2.01)	7.96 (−1.15)	8.76 (−0.85)
$SD[r_d]$	19.15	18.02 (−0.60)	17.52 (−0.86)	17.89 (−0.66)	17.42 (−0.91)	18.28 (−0.46)	17.50 (−0.87)
$SD[r_f]$	2.72	2.26 (−0.91)	3.05 (0.64)	2.61 (−0.22)	2.22 (−1.00)	3.10 (0.74)	2.61 (−0.22)
$SD[z_d]$	0.45	0.53 (1.22)	0.53 (1.20)	0.51 (0.93)	0.56 (1.72)	0.51 (0.90)	0.53 (1.28)
$AC[\Delta c]$	0.53	0.45 (−0.88)	0.43 (−1.07)	0.45 (−0.88)	0.46 (−0.79)	0.41 (−1.35)	0.44 (−0.96)
$AC[\Delta d]$	0.19	0.26 (0.68)	0.23 (0.32)	0.17 (−0.17)	0.26 (0.67)	0.23 (0.41)	0.18 (−0.12)
$AC[r_d]$	−0.01	−0.03 (−0.27)	0.02 (0.41)	−0.03 (−0.27)	0.02 (0.39)	0.02 (0.34)	0.01 (0.20)
$AC[r_f]$	0.68	0.69 (0.10)	0.72 (0.56)	0.72 (0.57)	0.65 (−0.51)	0.73 (0.73)	0.74 (0.83)
$AC[z_d]$	0.89	0.91 (0.50)	0.92 (0.67)	0.91 (0.36)	0.93 (0.82)	0.91 (0.45)	0.92 (0.60)
$Corr[\Delta c, \Delta d]$	0.54	0.51 (−0.14)	0.46 (−0.35)	0.52 (−0.10)	0.52 (−0.07)	0.45 (−0.43)	0.49 (−0.23)
$Corr[\Delta c, r_d]$	0.05	0.06 (0.20)	0.09 (0.58)	0.10 (0.69)	0.06 (0.12)	0.10 (0.73)	0.09 (0.58)
$Corr[\Delta d, r_d]$	0.07	0.22 (1.87)	0.14 (0.86)	0.08 (0.17)	0.22 (1.86)	0.14 (0.82)	0.09 (0.24)
$Corr[ep, z_{d,-1}]$	−0.16	−0.24 (−0.77)	−0.15 (0.17)	−0.15 (0.13)	−0.23 (−0.65)	−0.12 (0.42)	−0.14 (0.21)
$Corr[\Delta c, z_{d,-1}]$	0.19	0.65 (2.62)	0.65 (2.62)	0.63 (2.50)	0.66 (2.70)	0.63 (2.51)	0.64 (2.56)

Table 7.12: Extended long-run risk models. Data and average model-implied moments. t-statistic are in parentheses.

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