

The Matching Function and Nonlinear Business Cycles*

Online Appendix

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ABSTRACT

This appendix has three sections. First, it provides the derivation for the Nash bargaining equation and the proofs to the propositions and corollaries in the paper. Second, it includes a detailed description of our solution method. Third, it shows our results are robust to including home production, which allows for an alternative calibration of the outside employment option.

Keywords: Matching Function; Matching Elasticity; Nonlinear; Finding Rate; Unemployment

JEL Classifications: E24; E32; E37; J63; J64

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A DERIVATIONS AND PROOFS

A.1 WAGES To derive the wage rate under Nash bargaining, consider the household's problem:

$$J_t = \max_{c_t} c_t^{1-\gamma} / (1-\gamma) + \beta E_t J_{t+1}$$

subject to

$$c_t = w_t n_t + d_t + b u_t - \tau_t,$$

$$n_t = (1 - \bar{s}) n_{t-1} + f_t u_{t-1},$$

$$u_t = u_{t-1} + \bar{s} n_{t-1} - f_t u_{t-1},$$

where τ_t is a lump-sum tax and d_t are lump-sum dividends from firm ownership. The marginal values of employment and unemployment relative to the marginal utility of consumption are given by

$$\begin{aligned} J_{n,t}^H &= w_t + E_t [x_{t+1} ((1 - \bar{s}) J_{n,t+1}^H + \bar{s} J_{u,t+1}^H)], \\ J_{u,t}^H &= b + E_t [x_{t+1} (f_{t+1} J_{n,t+1}^H + (1 - f_{t+1}) J_{u,t+1}^H)]. \end{aligned}$$

Similarly, use the firm's problem to define the marginal value of employment to the firm,

$$J_{n,t}^F = a_t - w_t + (1 - \bar{s}) E_t [x_{t+1} J_{n,t+1}^F] = \frac{\kappa - \lambda_{v,t}}{q_t}.$$

Define the total surplus of a new match as $\Lambda_t = J_{n,t}^F + J_{n,t}^H - J_{u,t}^H$. The equilibrium wage maximizes $(J_{n,t}^H - J_{u,t}^H)^\eta (J_{n,t}^F)^{1-\eta}$. Optimality implies $J_{n,t}^H - J_{u,t}^H = \eta \Lambda_t$ and $J_{n,t}^F = (1 - \eta) \Lambda_t$. Combining the optimality conditions with $J_{n,t}^H$, $J_{u,t}^H$, and $J_{n,t}^F$, and defining tightness as $\theta_t = v_t / u_{t-1}$, we obtain

$$w_t = \eta (a_t + \kappa E_t [x_{t+1} \theta_{t+1}]) + (1 - \eta) b.$$

A.2 THE EFFICIENT ALLOCATION To solve for the efficient allocation, we imagine that the frictional labor market is controlled by a central planner who posts vacancies on behalf of firms, so it internalizes the two externalities associated with vacancy creation. The central planner solves

$$W_t = \max_{c_t, n_t, v_t} c_t^{1-\gamma} / (1-\gamma) + \beta E_t W_{t+1}$$

subject to

$$c_t = a_t n_t - \kappa v_t + b(1 - n_t) - \tau_t,$$

$$n_t = (1 - \bar{s}) n_{t-1} + \mathcal{M}(1 - n_{t-1}, v_t),$$

$$v_t \geq 0,$$

which imposes $u_t = 1 - n_t$. The efficient allocation is characterized by (1), (8), (10), and

$$\frac{\kappa - \lambda_{v,t}}{\mathcal{M}_v(1 - n_{t-1}, v_t)} = a_t - b + E_t[x_{t+1} \frac{\kappa - \lambda_{v,t+1}}{\mathcal{M}_v(1 - n_t, v_{t+1})} (1 - \bar{s} - \mathcal{M}_u(1 - n_t, v_{t+1}))], \quad (\text{A.1})$$

$$n_t = (1 - \bar{s})n_{t-1} + \mathcal{M}(1 - n_{t-1}, v_t). \quad (\text{A.2})$$

A.3 PROOFS Recall $\mathcal{M}(u_t^s, v_t)$ is strictly increasing, strictly concave, and twice differentiable in both arguments, and it exhibits constant returns to scale. We use the following standard results:

Lemma 1. $\mathcal{M}_{vv}(1, \theta_t)\theta_t = -\mathcal{M}_{uv}(1, \theta_t)$.

Lemma 2. *The elasticity of substitution has the equivalent representation*

$$\sigma(\theta_t) = \frac{\mathcal{M}_v(1, \theta_t)\mathcal{M}_u(1, \theta_t)}{\mathcal{M}_{vu}(1, \theta_t)\mathcal{M}(1, \theta_t)}.$$

Proposition 1 A constant returns to scale matching function, $\mathcal{M}(u_t^s, v_t)$, has linear approximation

$$\mathcal{M}(u_t^s, v_t) \approx \mathcal{M}(\bar{u}^s, \bar{v}) + \mathcal{M}_u(\bar{u}^s, \bar{v})(u_t^s - \bar{u}^s) + \mathcal{M}_v(\bar{u}^s, \bar{v})(v_t - \bar{v}),$$

where (\bar{u}^s, \bar{v}) is the point of approximation (e.g., a model's deterministic steady state). By constant returns to scale, Euler's theorem implies $\bar{m} \equiv \mathcal{M}(\bar{u}^s, \bar{v}) = \mathcal{M}_u(\bar{u}^s, \bar{v})\bar{u}^s + \mathcal{M}_v(\bar{u}^s, \bar{v})\bar{v}$. Combining these results and converting the steady-state partial derivatives into matching elasticities yields

$$\mathcal{M}(u_t^s, v_t) \approx (1 - \bar{\epsilon})\frac{\bar{m}}{\bar{u}^s}u_t^s + \bar{\epsilon}\frac{\bar{m}}{\bar{v}}v_t, \quad (\text{A.3})$$

where $\bar{\epsilon}$ is the matching elasticity evaluated at the approximation point. However, (A.3) is also the first-order approximation of a Cobb-Douglas matching function $\mathcal{M}(u_t^s, v_t) = \phi(u_t^s)^\alpha v_t^{1-\alpha}$ with $\alpha = 1 - \bar{\epsilon}$. Thus, using the Cobb-Douglas specification is without loss of generality up to first order.

Proposition 2 Differentiating the matching elasticity function $\epsilon(\theta_t) = \frac{\mathcal{M}_v(1, \theta_t)\theta_t}{\mathcal{M}(1, \theta_t)}$ yields

$$\epsilon'(\theta_t) = \left(\frac{\mathcal{M}_{vv}(1, \theta_t)\theta_t}{\mathcal{M}_v(1, \theta_t)} + 1 - \epsilon(\theta_t) \right) \frac{\mathcal{M}_v(1, \theta_t)}{\mathcal{M}(1, \theta_t)}.$$

Use Lemma 1 and Lemma 2 to obtain

$$\epsilon'(\theta_t) = \left(-\frac{1}{\sigma(\theta_t)} \frac{\mathcal{M}_u(1, \theta_t)}{\mathcal{M}(1, \theta_t)} + 1 - \epsilon(\theta_t) \right) \frac{\mathcal{M}_v(1, \theta_t)}{\mathcal{M}(1, \theta_t)}.$$

Replace $\frac{\mathcal{M}_u(1, \theta_t)}{\mathcal{M}(1, \theta_t)} = 1 - \epsilon(\theta_t)$ and rearrange to obtain

$$\epsilon'(\theta_t) = \frac{\sigma(\theta_t) - 1}{\sigma(\theta_t)} (1 - \epsilon(\theta_t)) \frac{\mathcal{M}_v(1, \theta_t)}{\mathcal{M}(1, \theta_t)}. \quad (\text{A.4})$$

Hence the sign of $\epsilon'(\theta_t)$ has the same sign as $\sigma(\theta_t) - 1$.

Corollary 1 Combine Proposition 2 with the fact that $\sigma(\theta_t) = \sigma$ for all $\theta_t > 0$.

Proposition 3 After imposing Assumption 1, (7) simplifies to

$$\frac{\kappa - \lambda_{v,t}}{q_t} = a_t - b + \beta(1 - \bar{s})E_t \left[\frac{\kappa - \lambda_{v,t+1}}{q_{t+1}} \right].$$

We can guess and verify a unique solution of the form $\frac{\kappa - \lambda_{v,t}}{q_t} = \delta_0 + \delta_1(a_t - \bar{a})$, where

$$\delta_0 = \frac{\bar{a} - b}{1 - \beta(1 - \bar{s})}, \quad \delta_1 = \frac{1}{1 - \beta(1 - \bar{s})\rho_a}.$$

If $\lambda_{v,t} > 0$ then $v_t = 0$. Since $m_t = v_t$ and $q_t = 1$ for v_t arbitrarily close to 0, we have $q_t = 1$ when $\lambda_{v,t} > 0$ by continuity. Therefore, if productivity is such that $\kappa/(\delta_0 + \delta_1(a_t - \bar{a})) \in [0, 1)$, then $q(a_t) = \kappa/(\delta_0 + \delta_1(a_t - \bar{a}))$ and $\lambda_{v,t} = 0$. Otherwise, $q_t = 1$ and $\lambda_{v,t} = \kappa - \delta_0 - \delta_1(a_t - \bar{a})$.

Proposition 4 Differentiate $\mu_q(\theta) = \mathcal{M}(1, \theta)/\theta$ to obtain $\mu'_q(\theta) = -\frac{1-\epsilon(\theta)}{\theta} \frac{\mathcal{M}(1, \theta)}{\theta}$. Hence

$$\theta'(a_t) = -\frac{q'(a_t)}{1 - \epsilon_t} \frac{\theta(a_t)^2}{\mathcal{M}(1, \theta(a_t))}.$$

Use $q'(a_t) = -q(a_t)^2 \delta_1 / \kappa$ and $q(a_t)\theta(a_t) = \mathcal{M}(1, \theta(a_t))$, to obtain

$$\theta'(a_t) = \frac{\delta_1}{\kappa} \frac{\mathcal{M}(1, \theta(a_t))}{1 - \epsilon_t} > 0.$$

Differentiate and use (A.4) to obtain

$$\theta''(a_t) = \frac{\delta_1}{\kappa} \frac{2\sigma_t - 1}{\sigma_t} \frac{\mathcal{M}_v(1, \theta(a_t))\theta'(a_t)}{1 - \epsilon_t}. \quad (\text{A.5})$$

Hence the sign of $\theta''(a_t)$ has the same sign as $\sigma_t - 1/2$.

Proposition 5 Differentiate $f'(a_t) = \mathcal{M}_v(1, \theta(a_t))\theta'(a_t)$ to obtain

$$f''(a_t) = \mathcal{M}_{vv}(1, \theta(a_t))\theta'(a_t)^2 + \mathcal{M}_v(1, \theta(a_t))\theta''(a_t).$$

Use Lemma 1 and Lemma 2 to obtain

$$f''(a_t) = \left(\theta''(a_t) - \frac{1}{\sigma(\theta)} \frac{\mathcal{M}_v(1, \theta)}{\mathcal{M}(1, \theta)} \frac{\theta'(a_t)^2}{\theta(a_t)} \right) \mathcal{M}_v(1, \theta(a_t)).$$

Replace $\frac{\mathcal{M}_v(1, \theta_t)}{\mathcal{M}(1, \theta_t)} = 1 - \epsilon(\theta_t)$ and use (A.5) to obtain

$$f''(a_t) = \left(\frac{\delta_1}{\kappa} \frac{2\sigma_t - 1}{\sigma_t} \frac{\mathcal{M}_v(1, \theta(a_t))}{1 - \epsilon_t} - \frac{1 - \epsilon_t}{\sigma_t} \frac{\theta'(a_t)}{\theta(a_t)} \right) \theta'(a_t) \mathcal{M}_v(1, \theta(a_t)).$$

Use $\theta'(a_t) = \frac{\delta_1}{\kappa} \frac{\mathcal{M}(1, \theta(a_t))}{1 - \epsilon_t}$ and $\epsilon_t = \frac{\mathcal{M}_v(1, \theta_t) \theta_t}{\mathcal{M}(1, \theta_t)}$ to obtain

$$f''(a_t) = \frac{2\sigma_t \epsilon_t - 1}{\sigma_t} \frac{\mathcal{M}_v(1, \theta(a_t)) (\theta'(a_t))^2}{\theta(a_t)}.$$

Hence the sign of $f''(a_t)$ is the same as the sign of $\sigma_t \epsilon_t - 1/2$.

Corollary 2 Recall that $q(a_t) \in (0, 1)$. When the matching function is CES, we have $\sigma_t = \sigma$ and $\epsilon_t = (1 - \vartheta)(\phi/q(a_t))^{(\sigma-1)/\sigma}$. By Proposition 5, the sign of $f''(a_t)$ depends on whether

$$\mathcal{F}_t \equiv 2\sigma(1 - \vartheta)(\phi/q(a_t))^{(\sigma-1)/\sigma} \lesseqgtr 1.$$

Case 1 ($\sigma > 1$): $(\phi/q(a_t))^{(\sigma-1)/\sigma} \in (\phi^{(\sigma-1)/\sigma}, \infty)$, so $\mathcal{F}_t > 2\sigma(1 - \vartheta)\phi^{(\sigma-1)/\sigma}$ for all feasible $q(a_t)$. Thus, $\sigma > \frac{1}{2(1-\vartheta)\phi^{(\sigma-1)/\sigma}} \geq 1$ implies $f''(a_t) > 0$ for all a_t such that $q(a_t) \in (0, 1)$.

Case 2 ($\sigma < 1$): $(\phi/q(a_t))^{(\sigma-1)/\sigma} \in (0, \phi^{(\sigma-1)/\sigma})$, so $\mathcal{F}_t < 2\sigma(1 - \vartheta)\phi^{(\sigma-1)/\sigma}$ for all feasible $q(a_t)$. Thus, $\sigma < \frac{1}{2(1-\vartheta)\phi^{(\sigma-1)/\sigma}} \leq 1$ implies $f''(a_t) < 0$ for all a_t such that $q(a_t) \in (0, 1)$.

Case 3 ($\sigma = 1$): $\sigma = 2(1 - \vartheta) = 1$ implies $f''(a_t) = 0$ for all a_t such that $q(a_t) \in (0, 1)$.

Corollary 3 Given the Den Haan et al. (2000) matching function, we have $\sigma_t = 1/(1 + \iota)$ and $\epsilon_t = q(a_t)^\iota$. By Proposition 5, the sign of $f''(a_t)$ depends on whether

$$\mathcal{F}_t = 2q(a_t)^\iota / (1 + \iota) \lesseqgtr 1.$$

Since $\iota > 0$, we have $2q(a_t)^\iota / (1 + \iota) < 2/(1 + \iota)$ for all feasible $q(a_t)$. Therefore $\iota > 1$ implies that $f''(a_t) < 0$ for all a_t such that $q(a_t) \in (0, 1)$.

Proposition 6 Given wedges $\{\tau_{v,t}, \tau_{n,t}\}$, the firm's optimal vacancy creation condition becomes

$$\frac{\kappa - \lambda_{v,t}}{q_t} = \frac{1 - \eta}{1 + \tau_{v,t}} (a_t - b) + E_t \left[\tilde{x}_{t+1} \frac{\kappa - \lambda_{v,t+1}}{q_{t+1}} \left(1 - \bar{s} - \frac{1}{1 + \tau_{v,t+1}} \frac{q_{t+1}}{\kappa - \lambda_{v,t+1}} (\kappa \eta \theta_{t+1} + \tau_{n,t+1}) \right) \right],$$

where $\tilde{x}_{t+1} \equiv x_{t+1}(1 + \tau_{v,t+1}) / (1 + \tau_{v,t})$. Setting

$$\begin{aligned} \tau_v(\theta_t) &= (1 - \eta) / \epsilon(\theta_t) - 1, \\ \tau_n(\theta_t) &= \theta_t ((\kappa - \lambda_{v,t}) \tau_v(\theta_t) - \eta \lambda_{v,t}), \end{aligned}$$

aligns the private optimality condition with the efficient condition (A.1). Differentiating yields

$$\begin{aligned} \tau_v'(\theta_t) &= -(1 - \eta) \epsilon'(\theta_t) / \epsilon(\theta_t)^2, \\ \tau_n'(\theta_t) &= \kappa (\theta_t \tau_v'(\theta_t) + \tau_v(\theta_t)) = \kappa \left[\frac{1 - \eta}{\epsilon(\theta_t)} - 1 - \frac{1 - \eta}{\epsilon(\theta_t)^2} \theta_t \epsilon'(\theta_t) \right]. \end{aligned}$$

Since (A.4) implies $\epsilon'(\theta_t)\theta_t/\epsilon(\theta_t) = (\sigma_t - 1)(1 - \epsilon(\theta_t))/\sigma_t$, we obtain

$$\begin{aligned}\tau'_v(\theta_t) &= - \left(\frac{1 - \eta}{\theta_t} \right) \left(\frac{\sigma_t - 1}{\sigma_t} \right) \left(\frac{1 - \epsilon(\theta_t)}{\epsilon(\theta_t)} \right), \\ \tau'_n(\theta_t) &= \kappa \left[\frac{1 - \eta}{\epsilon(\theta_t)} - 1 - (1 - \eta) \left(\frac{\sigma_t - 1}{\sigma_t} \right) \left(\frac{1 - \epsilon(\theta_t)}{\epsilon(\theta_t)} \right) \right].\end{aligned}$$

Hence, $\tau'_v(\theta_t) > 0$ when $\sigma_t < 1$ and $\tau'_n(\theta_t) > 0$ when $\sigma_t < \frac{1-\eta}{\eta} \frac{1-\epsilon_t}{\epsilon_t}$ for all $\theta_t > 0$.

B SOLUTION METHOD

The equilibrium system of the model is summarized by $E[g(\mathbf{x}_{t+1}, \mathbf{x}_t, \varepsilon_{t+1}) | \mathbf{z}_t, \mathcal{P}] = 0$, where g is a vector-valued function, \mathbf{x}_t is a vector of variables, ε is a vector of productivity shocks, \mathbf{z}_t is a vector of states, and \mathcal{P} is a vector of parameters. There are many ways to discretize the productivity process. We use the Markov chain in Rouwenhorst (1995), which Kopecky and Suen (2010) show outperforms other methods for approximating autoregressive processes. The bounds on the state variable n_{t-1} are set to $[0.85, 0.98]$, which contains over 99% of the ergodic distribution. We discretize a_t and n_{t-1} into 7 and 21 evenly-spaced points, respectively. The product of the points in each dimension, D , is the total nodes in the state space ($D = 147$). The realization of \mathbf{z}_t on node d is denoted $\mathbf{z}_t(d)$. The Rouwenhorst method provides integration weights, $\phi(m)$, for $m \in \{1, \dots, M\}$.

Since vacancies $v_t \geq 0$, we introduce an auxiliary variable, μ_t , such that $v_t = \max\{0, \mu_t\}^2$ and $\lambda_{0,t} = \max\{0, -\mu_t\}^2$, where $\lambda_{v,t}$ is the Lagrange multiplier on the non-negativity constraint. If $\mu_t \geq 0$, then $v_t = \mu_t^2$ and $\lambda_{v,t} = 0$. When $\mu_t < 0$, the constraint is binding, $v_t = 0$, and $\lambda_{v,t} = \mu_t^2$. Therefore, the constraint on v_t is transformed into a pair of equalities (Garcia and Zangwill, 1981).

The following steps outline our nonlinear policy function iteration algorithm:

1. Use Sims's (2002) `gensys` algorithm to solve the linearized model. Then map the solution for the policy functions to the discretized state space. This provides an initial conjecture.
2. On iteration $j \in \{1, 2, \dots\}$ and each node $d \in \{1, \dots, D\}$, use Chris Sims's `csolve` to find $\mu_t(d)$ to satisfy $E[g(\cdot) | \mathbf{z}_t(d), \mathcal{P}] \approx 0$. Guess $\mu_t(d) = \mu_{j-1}(d)$. Then apply the following:
 - (a) Solve for all variables dated at time t , given $\mu_t(d)$ and $\mathbf{z}_t(d)$.
 - (b) Linearly interpolate the policy function, μ_{j-1} , at the updated state variables, $\mathbf{z}_{t+1}(m)$, to obtain $\mu_{t+1}(m)$ on every integration node, $m \in \{1, \dots, M\}$.
 - (c) Given $\{\mu_{t+1}(m)\}_{m=1}^M$, solve for the other elements of $\mathbf{x}_{t+1}(m)$ and compute

$$E[g(\mathbf{x}_{t+1}, \mathbf{x}_t(d), \varepsilon_{t+1}) | \mathbf{z}_t(d), \mathcal{P}] \approx \sum_{m=1}^M \phi(m) g(\mathbf{x}_{t+1}(m), \mathbf{x}_t(d), \varepsilon_{t+1}(m)).$$

Set $\mu_j(d) = \mu_t(d)$ when `csolve` converges.

3. Repeat step 2 until $\text{maxdist}_j < 10^{-7}$, where $\text{maxdist}_j \equiv \max\{|\mu_j - \mu_{j-1}|\}$. When that criterion is satisfied, the algorithm has converged to an approximate nonlinear solution.

The algorithm is programmed in Fortran with Open MPI and run on the BigTex supercomputer.

C HOME PRODUCTION

In the baseline model, we set b to target the standard deviation of unemployment in our sample. This section shows we can equivalently set b externally as an unemployment benefit, and instead use home production to target unemployment volatility by following Petrosky-Nadeau et al. (2018).

The household derives utility from the consumption of the final market good $c_{m,t}$ and home production $c_{h,t}$. It has log utility over composite consumption $c_t = (\omega c_{m,t}^e + (1 - \omega)c_{h,t}^e)^{1/e}$, where $\omega \in (0, 1)$ is the preference weight on the final market good and $e \leq 1$ governs the elasticity of substitution $1/(1 - e)$. The home production technology is $c_{h,t} = a_h u_t$, where $a_h > 0$ is productivity.

Household optimization yields the pricing kernel $x_{t+1} = \beta(c_{m,t}/c_{m,t+1})^{1-e}(c_t/c_{t+1})^e$. The flow value of unemployment becomes $z_t = a_h((1 - \omega)/\omega)(c_{m,t}/c_{h,t})^{1-e} + b$, so the Nash wage satisfies

$$w_t = \eta((1 - \alpha)y_t/n_t + \kappa(1 - \chi\bar{s})E_t[x_{t+1}\theta_{t+1}]) + (1 - \eta)z_t.$$

The other equilibrium conditions are unchanged from the baseline model described in Section 3.

We set $b = 0.4$ to reflect the value of unemployment benefits (Shimer, 2005), and set $a_h = 1$ to steady-state labor productivity in final good production. We then set $e = 1$, in line with existing calibrations and estimates (Benhabib et al., 1991; Petrosky-Nadeau et al., 2018). In this case, $z_t = (1 - \omega)/\omega + b$, so ω determines the level of z , and hence the volatility of unemployment following the fundamental surplus arguments in Ljungqvist and Sargent (2017). Thus, we can set ω in each model to generate the same unemployment volatility and quantitative results as the baseline model.

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